A Generalized Model for Cost and Fairness Analysis in Coded Cooperative Data Exchange

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- ✓ Fairness
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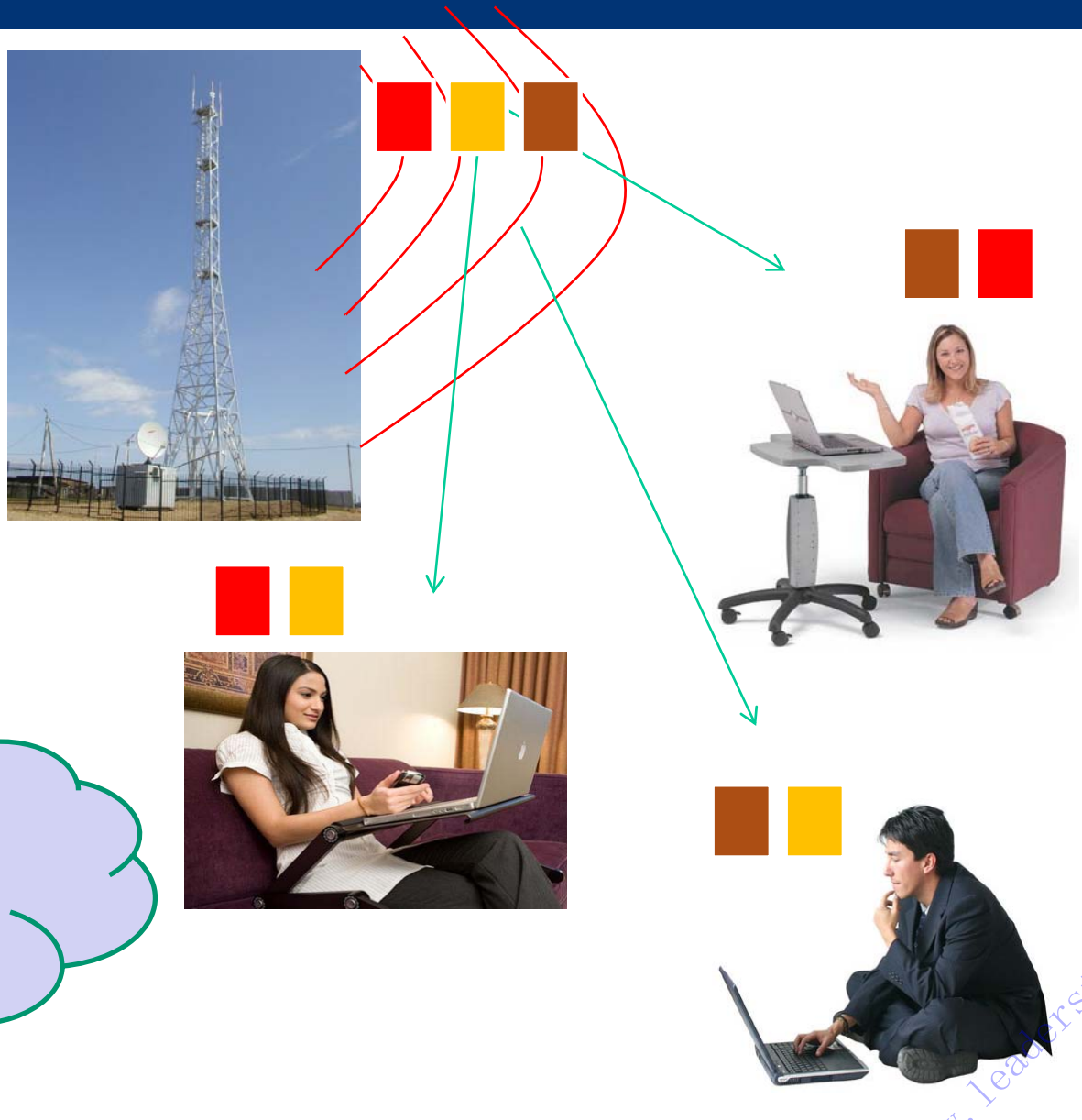
Introduction

Cooperative Data Exchange

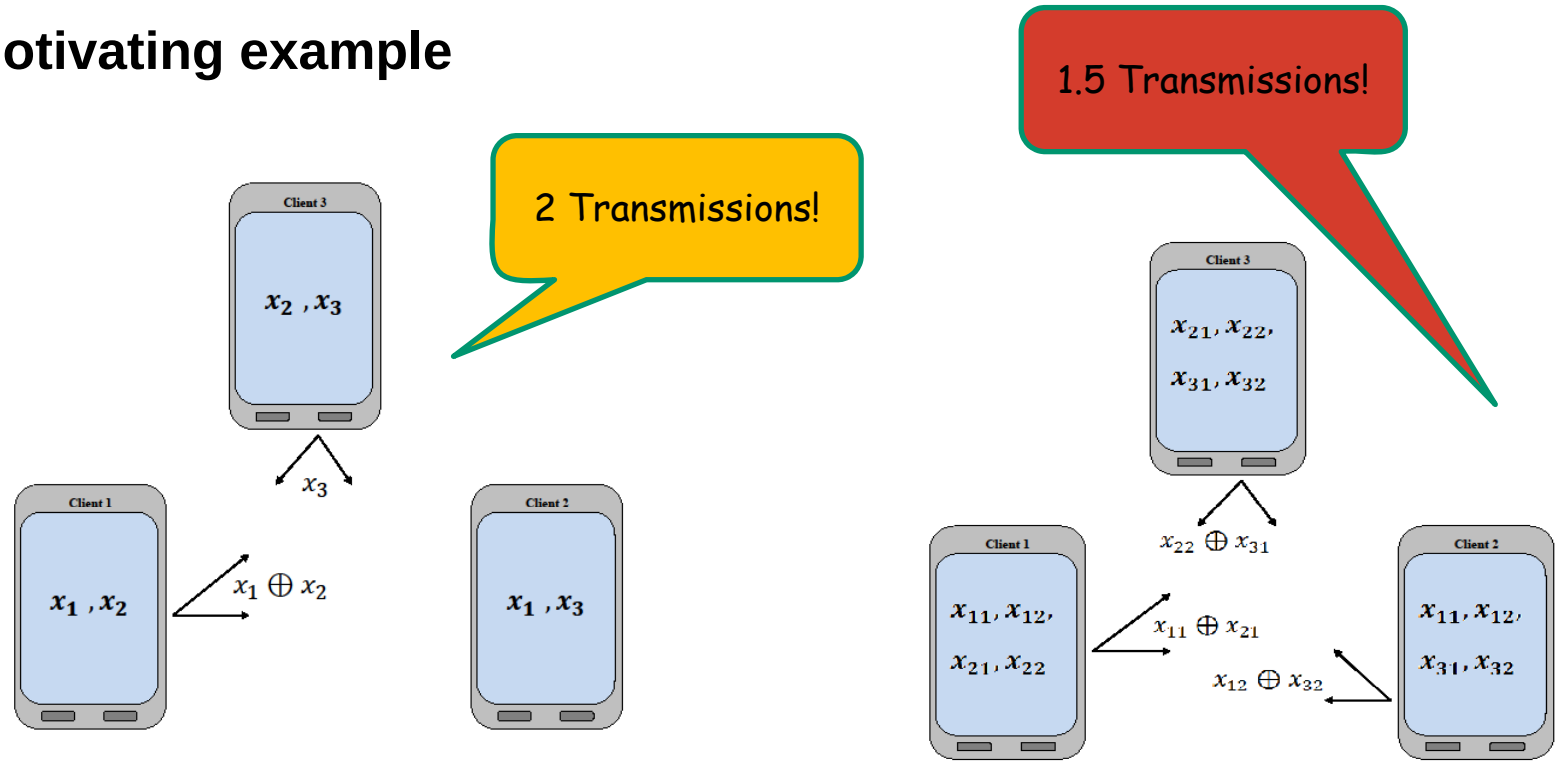
Retransmission by the server or Cooperation by the clients? This is the problem!



How many packets should be exchanged?

Introduction

A motivating example



$x_i \in GF(2^m)$ 1 1 0 1 0 1 $\Rightarrow$ $x_{i1}, x_{i2} \in GF(2^{m/2})$ 1 1 0 1 0 1

Related Work

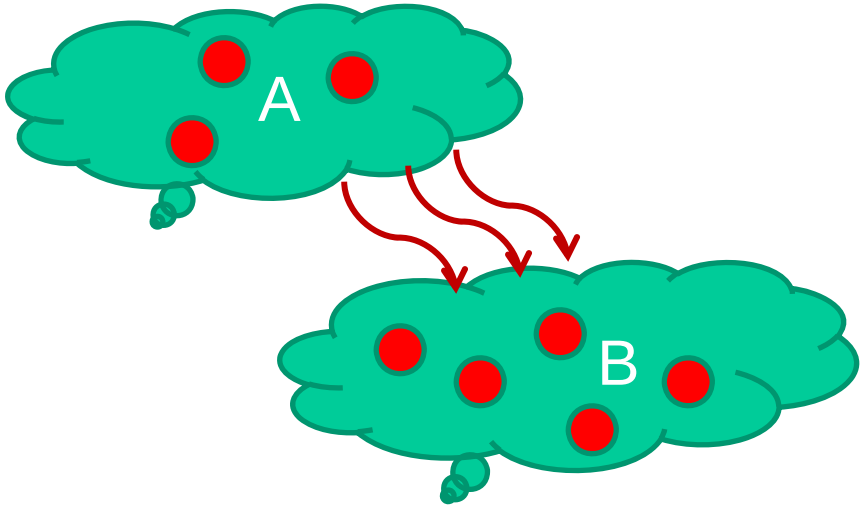
- ✓ Heuristics
  - Delay (Liu et al., 2008)
  - Total number of transmissions

    (Sprinston, Sadeghi, et al., ISIT,2010, ITW 2010)
  - Minimum Cost

    (Ozgul et al., 2011)
- ✓ Exact Solutions
  - Linear Programming Model

    (E. Tajbakhsh, et al., 2010, Courtade et al. 2010)
  - Network Flow Model

    Current paper



$$min \sum z_i$$

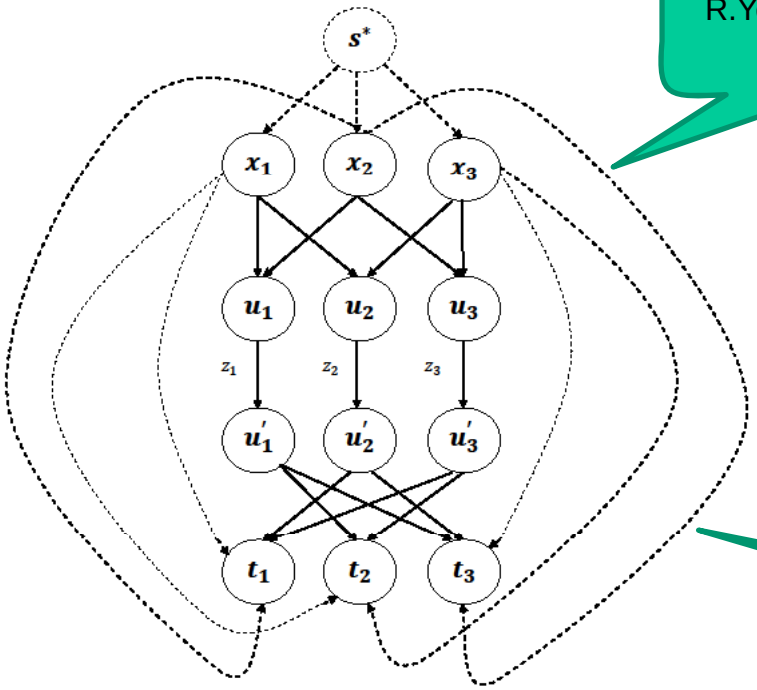
S.T.

$$\sum_{i \in A} z_i \geq |\bar{X}_B|$$

Optimal Solution (cont'd)

Linear Programming Formulation

R.Yeung & Z. Zhang, 1999.



$$\text{minimize} \quad \sum_{i=1}^{k} b_i z_i$$

$$\text{subject to}$$

$$0 \le \alpha_e^{t_i} \le z_i \le C_i$$

$$\sum_{\Gamma_o(v)} \alpha_e^{t_i} - \sum_{\Gamma_i(v)} \alpha_e^{t_i} = \sigma_v^{(t_i)}$$

where,

$$\sigma_v^{(t_i)} = \begin{cases} R, & \text{if } v = s^*, \\ -R, & \text{if } v = t_i, \\ 0, & \text{otherwise.} \end{cases}$$

Bakshi & Effros, 2008.

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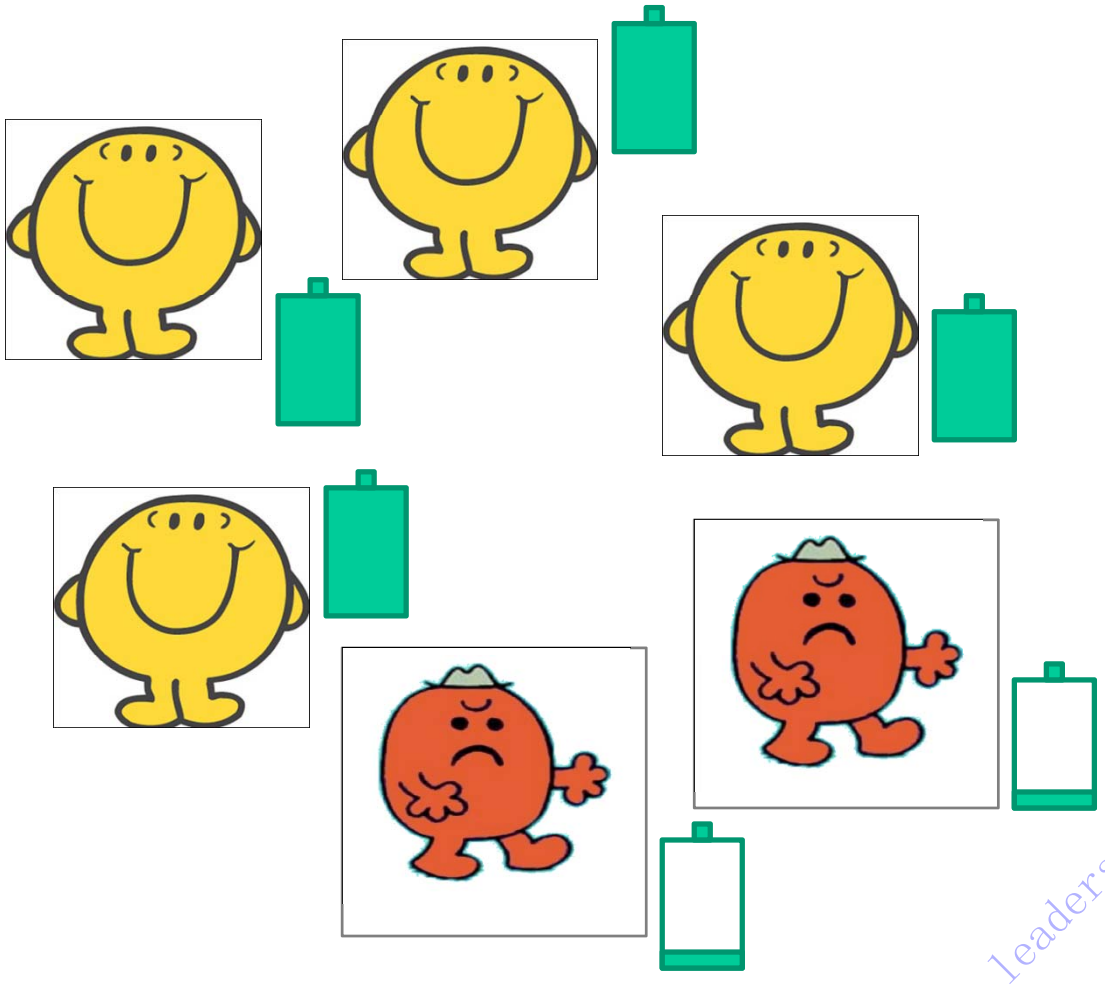
Fairness

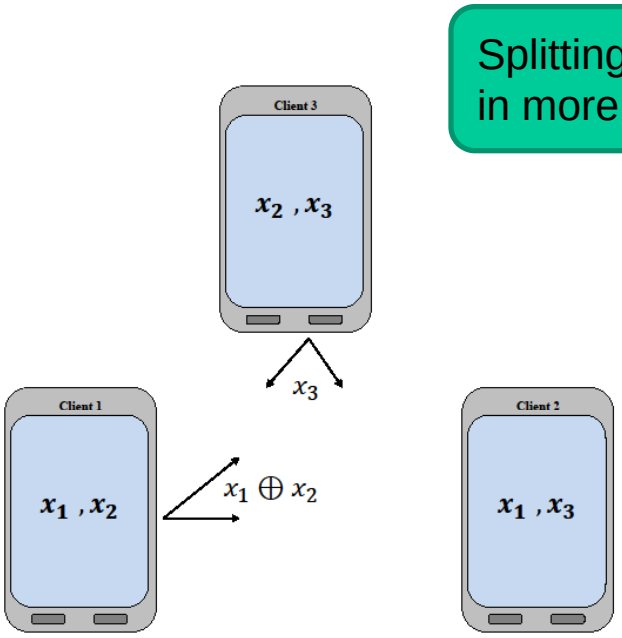
✓ Fairness is a critical issue when the energy supplies of transceiver devices are limited.

✓ A quantitative measure of fairness should be continuous, independent of scale and unit, and within a limited range. The metric suggested by Jain et al. (1984) holds all the mentioned properties:

$$f(\mathbf{z}) = \frac{(\sum_{i=1}^{k} z_i)^2}{k \sum_{i=1}^{k} {z_i}^2}$$

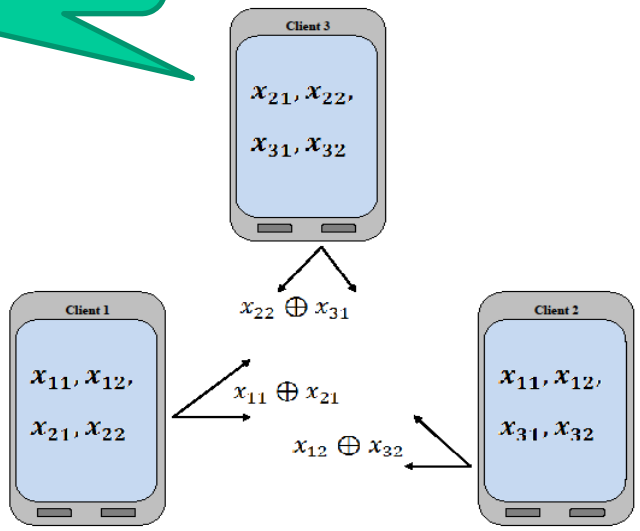


Fairness (cont'd)



$$f(z) = \frac{2}{3}$$

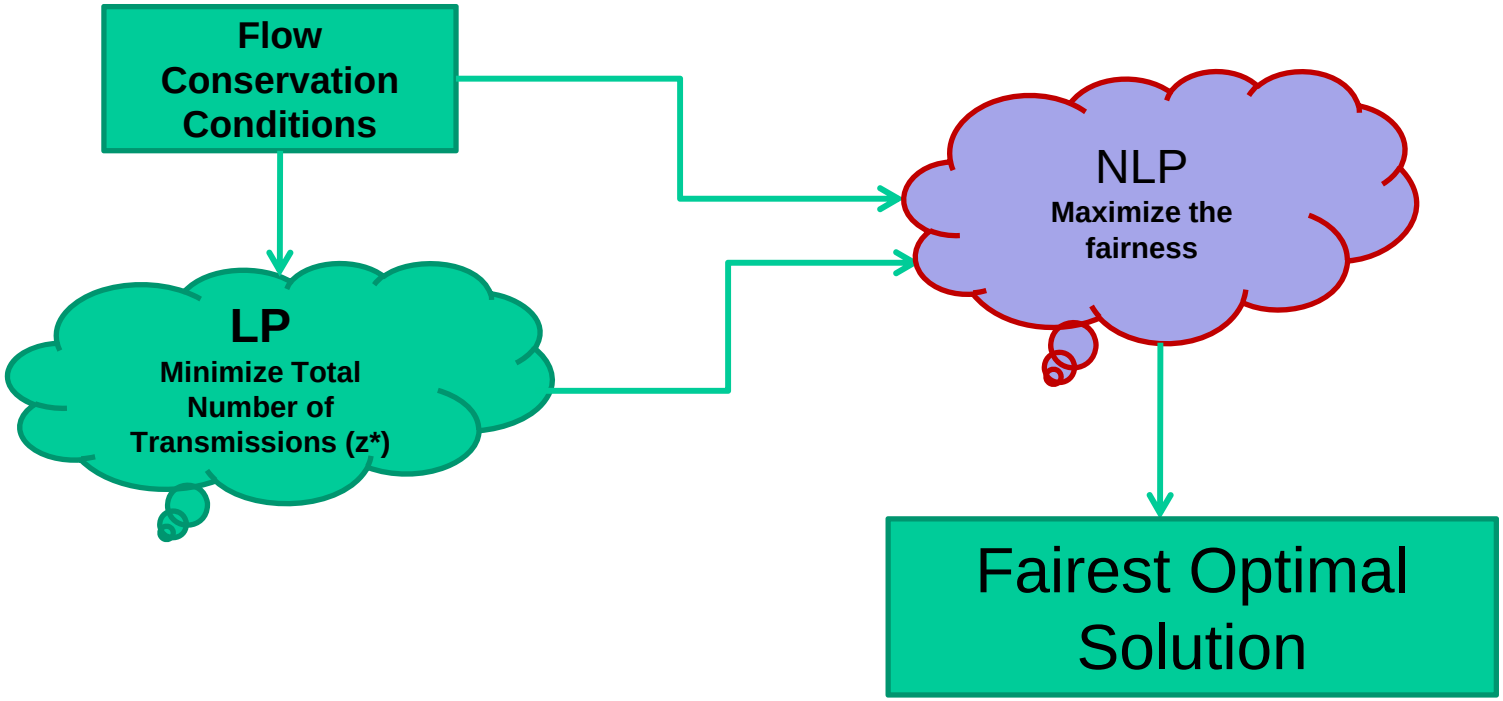
Splitting the packets results in more fairness



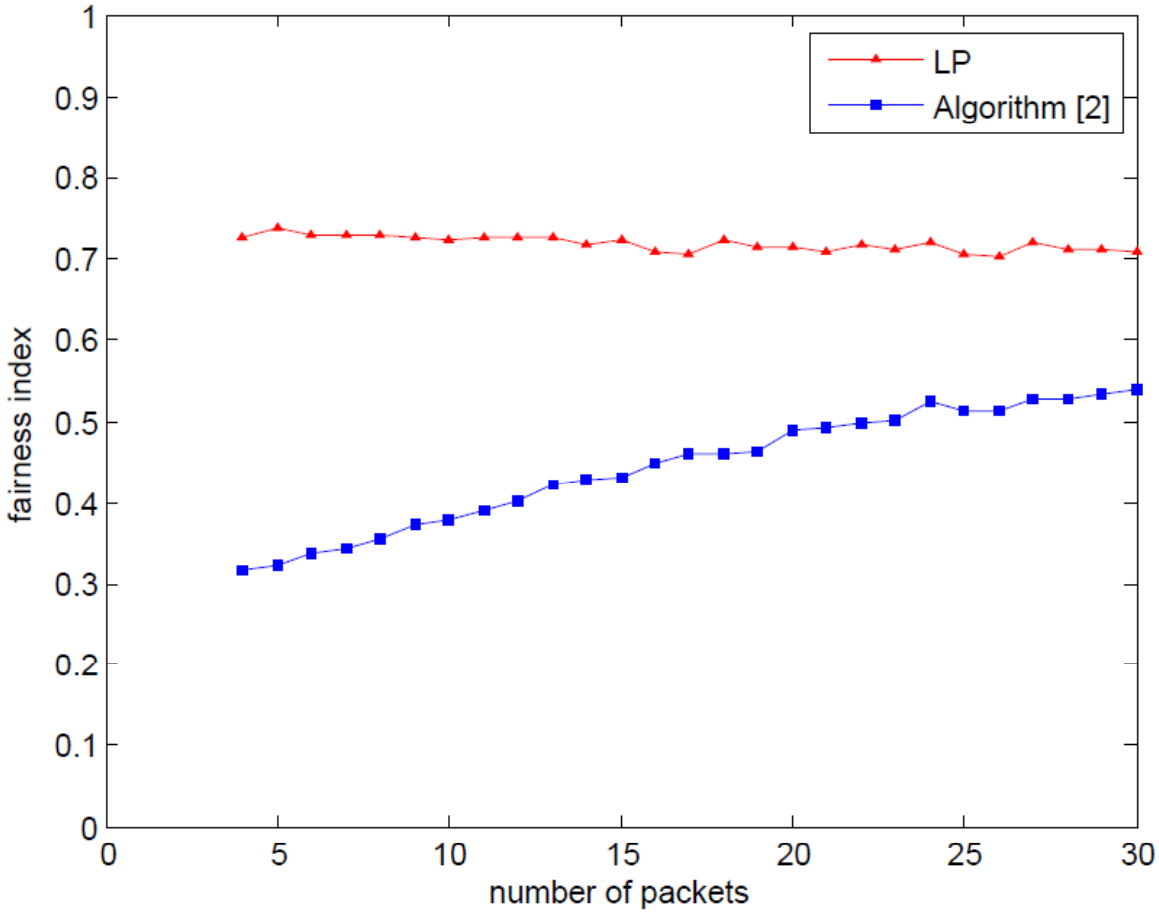
$$f(z) = 1$$

Fairness (cont'd)

Fine Tuning the Fairness

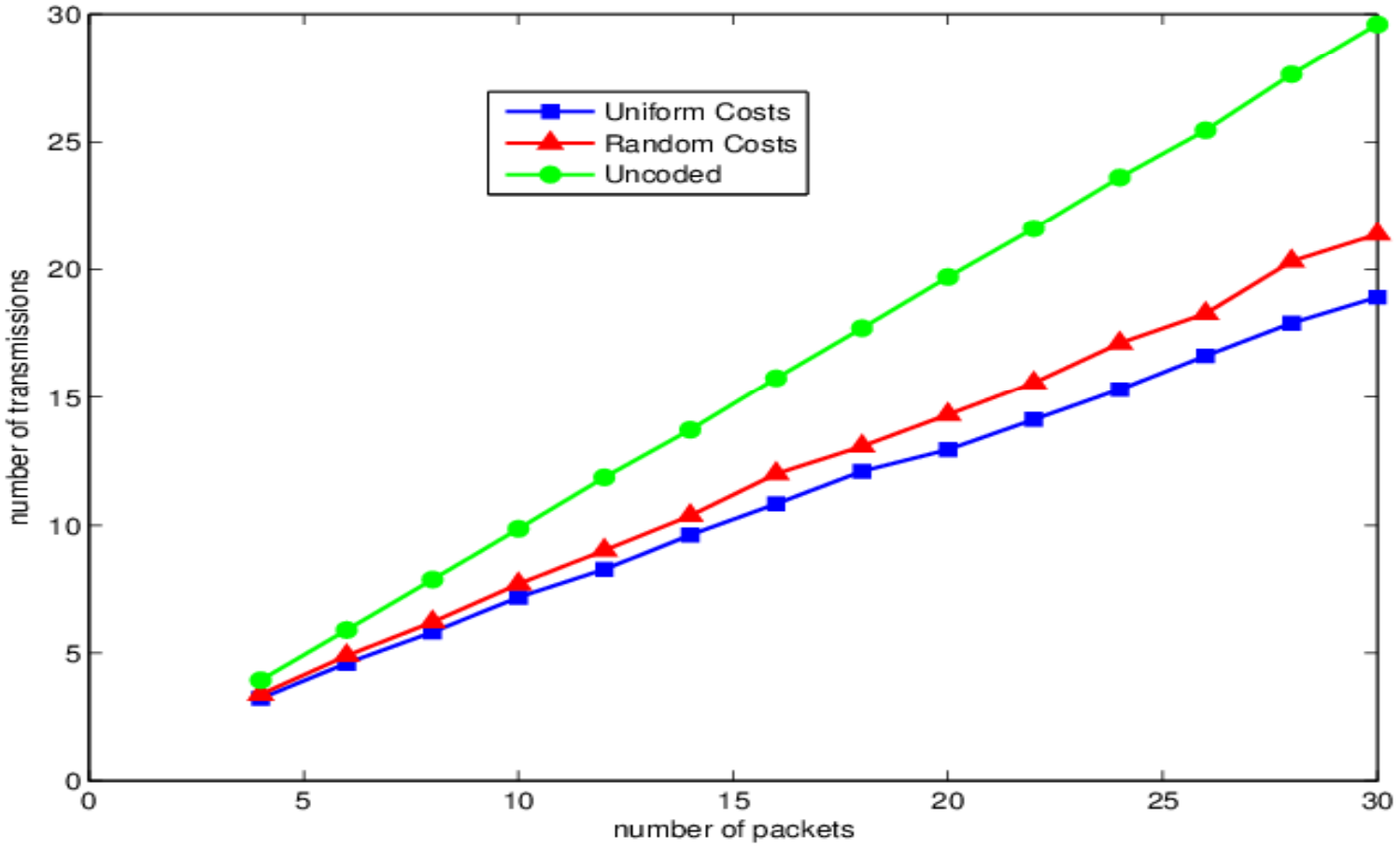


Numerical Experiments



Splitting the packets: Dramatic Improvement in Fairness

Numerical Experiments (cont'd)



Conclusion:

- ✓ The Cooperative Data Exchange problem can be modeled as a multicast with side information at the sinks.
- ✓ The minimum cost solution can be found using a min cost network flow problem if the appropriate model is used.
- ✓ Naturally, solving the LP results in fractional numbers interpreted as splitting the packets in practice.
- ✓ When the solution to the LP is not unique it is meaningful to think about finding the fairest solution among the set of optimal solutions.

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