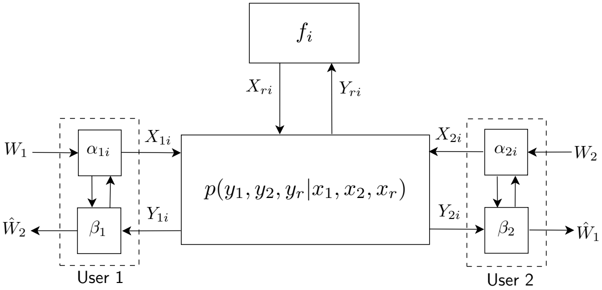
Hybrid Digital–Analog Noisy Network Coding

Majid Nasiri Khormuji and Mikael Skoglund

NetCod 2011

The Two-Way Relay Channel (TWRC)



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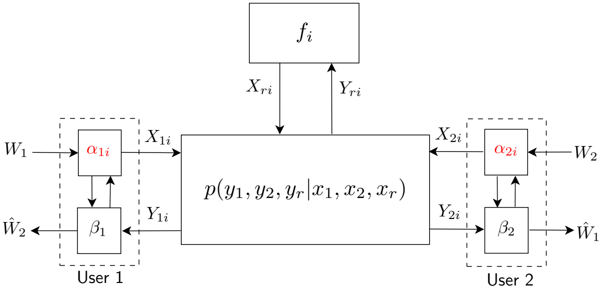

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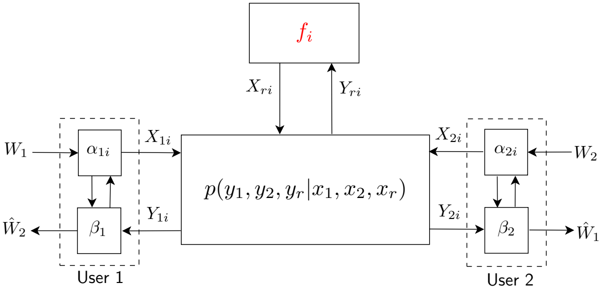

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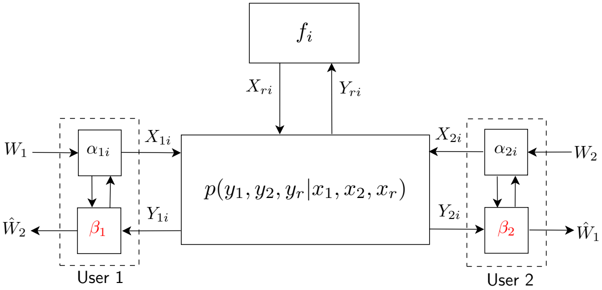

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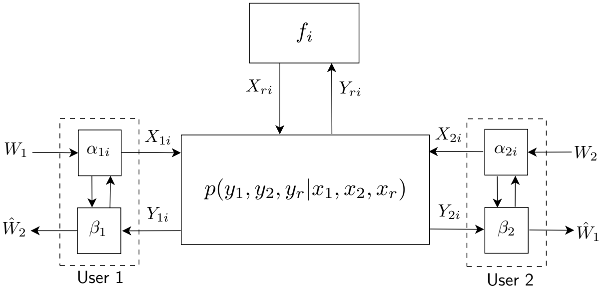

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- Decoding: $\hat{W}_1 = \beta_2(Y_{21}^n, W_2)$ and $\hat{W}_2 = \beta_1(Y_{11}^n, W_1)$.
- Achievable Rate: The rate pair (R_1, R_2) is achievable if there is a 5-tuple $(\{\alpha_{1i}\}_{i=1}^n, \{\alpha_{2i}\}_{i=1}^n, \{f_i\}_{i=1}^n, \beta_1, \beta_2)$ such that the average message error probability $P_e^{(n)} = \Pr\{(W_1, W_2) \neq (\hat{W}_1, \hat{W}_2)\} \to 0$ as $n \to \infty$ and $\liminf_{n\to\infty} \frac{1}{n} \log M_j(n) \geq R_j, \quad j \in \{1, 2\}$

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- TWRC is a fairly well-studied channel model. However, the complete characterization of the capacity remains open.
- Here, we focus on compression-based schemes such as compress-and-forward (CF) and noisy network coding (NNC), and present three hybrid digital–analog (HDA) schemes.
- We demonstrate that the proposed HDA schemes outperform purely digital and purely analog schemes.

Scheme 1

- Core idea: The relay generates a compression codebook using the Wyner-Ziv principle for one user and then it combines the encoded compression index with the noisy analog signal. In the scheme, one user *only* recovers the digital signal and the other treats the digital signal as a noise. Shannon strategy is used to combine the digital and analog signal.

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- The rate region $\mathcal{R}_{\mathrm{H}_1} = \mathcal{R}_1 \bigcup \mathcal{R}_2$ is achievable where

$$\mathcal{R}_1 = \Big\{ (R_1, R_2) : R_1 \leq I(X_1; Y_2, \hat{Y}_r | X_2, V), R_2 \leq I(X_2; Y_1 | X_1), \\ I(Y_r; \hat{Y}_r | X_2, V, Y_2) \leq I(V; Y_2 | X_2) \Big\} \quad (1)$$

$$\mathcal{R}_2 = \Big\{ (R_1, R_2) : R_1 \leq I(X_1; Y_2 | X_2), R_2 \leq I(X_2; Y_1, \hat{Y}_r | X_1, V), \\ I(Y_r; \hat{Y}_r | X_1, V, Y_1) \leq I(V; Y_1 | X_1) \Big\} \quad (2)$$

for some $p(x_1)p(x_2)p(v)p(\hat{y}_r | y_r, x_r), x_r = f(y_r, v)$.

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- The rate region

$$\begin{aligned}\mathcal{R}_{\mathrm{CF}} = \Big\{(R_1, R_2) : R_1 &\leq I(X_1; Y_2, \hat{Y}_r | X_2, V), \\ R_2 &\leq I(X_2; Y_1, \hat{Y}_r | X_1, V), \\ &\max\Big(I(Y_r; \hat{Y}_r | X_1, V, Y_1), I(Y_r; \hat{Y}_r | X_2, V, Y_2)\Big) \\ &\leq \min\left(I(V; Y_1 | X_1), I(V; Y_2 | X_2)\right)\Big\}, \qquad (3)\end{aligned}$$

is achievable for some $p(x_1)p(x_2)p(v)p(\hat{y}_r|y_r, v)$ and $x_r = f(v, y_r)$.

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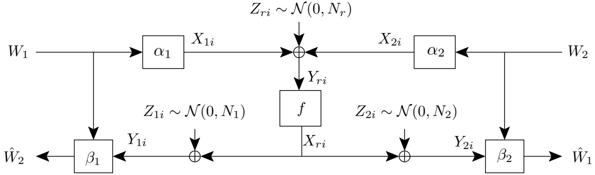
The Gaussian Two-Way Relay Channel



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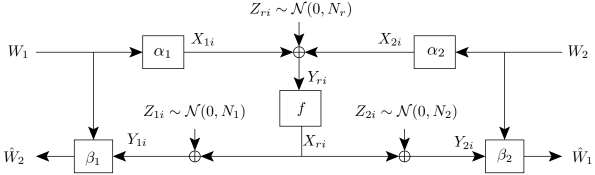

- The users transmit symbols X_{1i} and X_{2i} and the relay receives $Y_{ri} = X_{1i} + X_{2i} + Z_{ri}$ where $Z_{ri} \sim \mathcal{N}(0, N_r)$ is the additive Gaussian noise at the relay.

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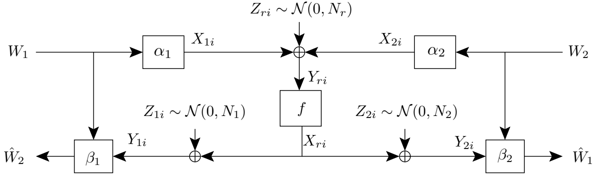

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- The user $j \in \{1, 2\}$ then receives $Y_{ji} = X_{ri} + Z_{ji}$ where X_{ri} is the transmitted symbol from the relay and $Z_{ji} \sim \mathcal{N}(0, N_j)$ denotes the additive noise at the jth user.

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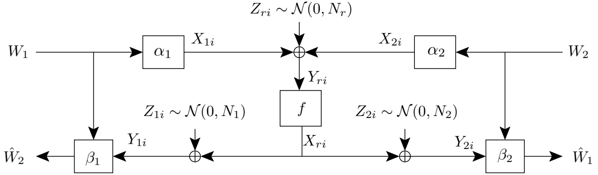

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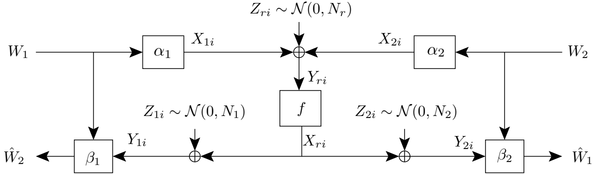

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- We assume that all noise sequences are white and mutually independent from each other.
- We additionally assume that the users and the relay operate under average power constraints, i.e. $\sum_{i=1}^{n} \mathbb{E}\left[X_{ri}^2\right] \leq nP_r$, $\sum_{i=1}^{n} \mathbb{E}\left[X_{ji}^2\right] \leq nP_j$ for $j \in \{1, 2\}$.

Hybrid Relaying

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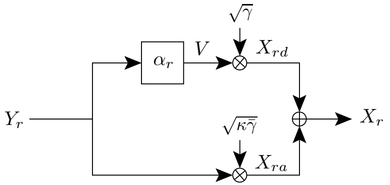
- Let $X_1 \sim \mathcal{N}(0, P_1)$, $X_2 \sim \mathcal{N}(0, P_2)$ and $V \sim \mathcal{N}(0, \gamma P_r)$. Then let $\hat{Y}_r = Y_r + Z_q$ where $Z_q \sim \mathcal{N}(0, N_q)$ is independent of other random variables and

$$X_r = X_{rd} + X_{ra} = \sqrt{\gamma} V + \sqrt{\kappa\bar{\gamma}} Y_r$$

where γ is a power splitting parameter and $\kappa = P_r(P_1 + P_2 + N_r)^{-1}$ is a power normalization factor.



Hybrid Relaying: Scheme 1

The rate region $\mathcal{R}_{\mathrm{H}_1} = \mathcal{R}_1 \bigcup \mathcal{R}_2$ is achievable where

$$\mathcal{R}_1 = \bigcup_{\gamma \in [0,1]} \Bigg\{ (R_1, R_2) : R_2 \le \mathcal{C}\left(\frac{\bar{\gamma}\kappa P_2}{\gamma P_r + \bar{\gamma}\kappa N_r + N_1} \right), \quad R_1 \le \mathcal{C}\left(\frac{P_1 N_2 + \bar{\gamma}\kappa P_1 N_q}{N_r N_2 + \bar{\gamma}\kappa N_r N_q + N_2 N_q} \right), \quad N_q = \frac{(P_1 + N_r)N_2}{\gamma P_r} \Bigg\}, \tag{5}$$

$$\mathcal{R}_2 = \bigcup_{\gamma \in [0,1]} \Bigg\{ (R_1, R_2) : R_1 \le \mathcal{C}\left(\frac{\bar{\gamma}\kappa P_1}{\gamma P_r + \bar{\gamma}\kappa N_r + N_2} \right), \quad R_2 \le \mathcal{C}\left(\frac{P_2 N_1 + \bar{\gamma}\kappa P_2 N_q}{N_r N_1 + \bar{\gamma}\kappa N_r N_q + N_1 N_q} \right), \quad N_q = \frac{(P_2 + N_r)N_1}{\gamma P_r} \Bigg\}, \tag{6}$$

$\bar{\gamma} := 1 - \gamma$ and $\mathcal{C}(x) := 0.5 \log_2(1 + x)$.

Hybrid Relaying: Scheme 2

The rate region $\mathcal{R}_{\mathrm{H}_2}$ is achievable where

$$\mathcal{R}_{\mathrm{H}_2} = \bigcup_{\gamma\in[0,1]} \Bigg\{ (R_1, R_2) : R_1 \leq \mathcal{C}\left(\frac{P_1 N_2 + \bar{\gamma}\kappa P_1 N_q}{N_r N_2 + \bar{\gamma}\kappa N_r N_q + N_2 N_q}\right),$$

$$R_2 \leq \mathcal{C}\left(\frac{P_2 N_1 + \bar{\gamma}\kappa P_2 N_q}{N_r N_1 + \bar{\gamma}\kappa N_r N_q + N_1 N_q}\right),$$

$$N_q = \frac{1}{\gamma P_r} \max\left(\frac{(P_1 + N_r)N_2}{N_2 + \bar{\gamma}\kappa(P_1 + N_r)}, \frac{(P_2 + N_r)N_1}{N_1 + \bar{\gamma}\kappa(P_2 + N_r)}\right)$$

$$\times \max\left(N_1 + \bar{\gamma}\kappa(P_2 + N_r), N_2 + \bar{\gamma}\kappa(P_1 + N_r)\right) \Bigg\}. \quad (7)$$

Hybrid Relaying: Scheme 3

The rate region $\mathcal{R}_{\mathrm{H}_3}$ is achievable where

$$\mathcal{R}_{\mathrm{H}_3} = \bigcup_{\gamma\in[0,1]} \Bigg\{ (R_1, R_2) : R_1 \le \mathcal{C}\left(\frac{\gamma P_r + \kappa\bar{\gamma}P_1}{\kappa\bar{\gamma}N_r + N_2}\right) - \mathcal{C}\left(\frac{N_r N_2}{N_q(\bar{\gamma}\kappa N_r + N_2)}\right),$$
$$R_2 \le \mathcal{C}\left(\frac{\gamma P_r + \kappa\bar{\gamma}P_2}{\kappa\bar{\gamma}N_r + N_1}\right) - \mathcal{C}\left(\frac{N_r N_1}{N_q(\bar{\gamma}\kappa N_r + N_1)}\right),$$
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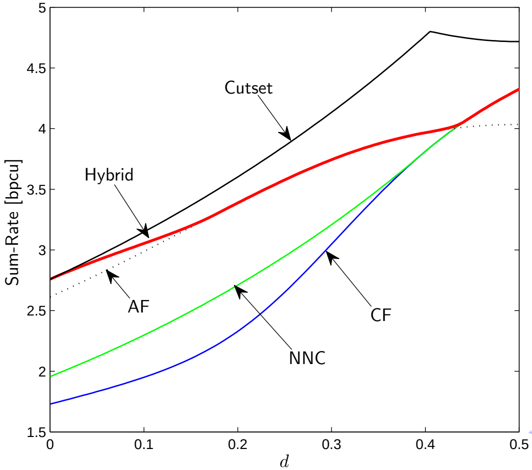
$$R_2 \le \mathcal{C}\left(\frac{P_2 N_1 + \bar{\gamma}\kappa P_2 N_q}{N_r N_1 + \bar{\gamma}\kappa N_r N_q + N_1 N_q}\right), N_q > 0 \Bigg\}. \tag{8}$$

Performance Comparison

- We assume that User 1, User 2 and the relay are located on a straight line, where the distance of User 1 to the relay is $d \in [0, 1]$ and the distance of the relay to User 2 is $1 - d$.
- In order to take into account the geometry of the network, we let the channel parameters be $P_1 = Pd^{-3}$, $P_2 = P(1-d)^{-3}$, $N_1 = d^3$, $N_2 = (1-d)^3$ and $N_r = 1$.

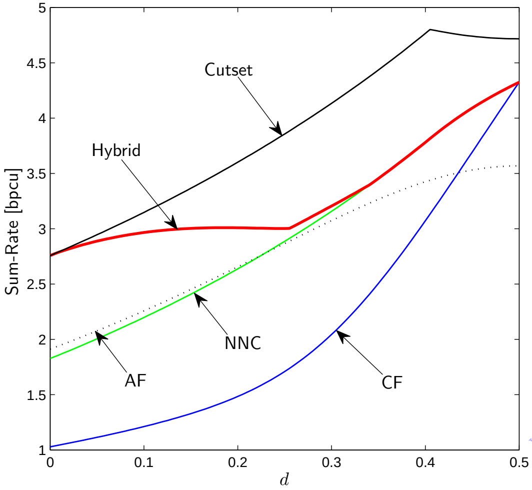
Numerical Examples (1)

$P = 5$ and $P_r = 10$ dB.



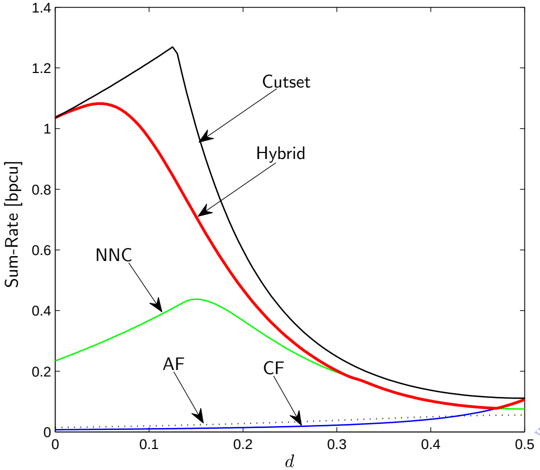
Numerical Examples (2)

$P = 10$ and $P_r = 5$ dB.



Numerical Examples (3)

$P = 5$ and $P_r = -20$ dB.



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- We presented an achievable rate regions based on hybrid digital–analog NNC.
- Hybrid NNC outperform NNC, CF and AF.
- For the geometric Gaussian network, the capacity region when $d \longrightarrow 0$ is given by

$$\mathcal{R} = \left\{ (R_1, R_2) : R_1 \leq \mathcal{C}\left(\frac{P_r}{N_2}\right), R_2 \leq \mathcal{C}\left(\frac{P_2}{N_r}\right) \right\} \tag{9}$$

where the capacity is achieved by Scheme 1.

Questions

Thank You!