Soft-Information-Based Joint Network-Channel Coding for the Two-Way Relay Channel

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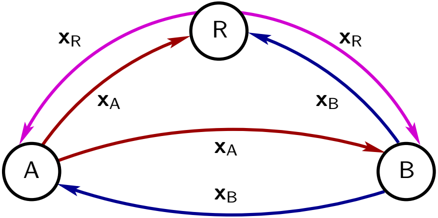
Outline

1. Two-way relay channel
2. Proposed scheme
3. Simulation results
4. Conclusions and outlook

- **Two-way relay channel (TWRC)**
  - A and B exchange independent messages
  - relay R supports transmission A $\leftrightarrow$ B
  - **large variety of settings and coding strategies**



- **Goal**: devise a coding scheme for the TWRC with
  - excellent **performance**
  - low **complexity** and **delay**
  - considering practical constraints

Introduction

- **Two-way relay channel (TWRC)**
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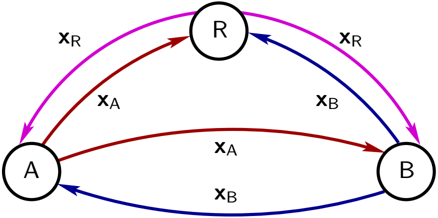
  - relay R supports transmission A $\leftrightarrow$ B
  - **large variety of settings and coding strategies**



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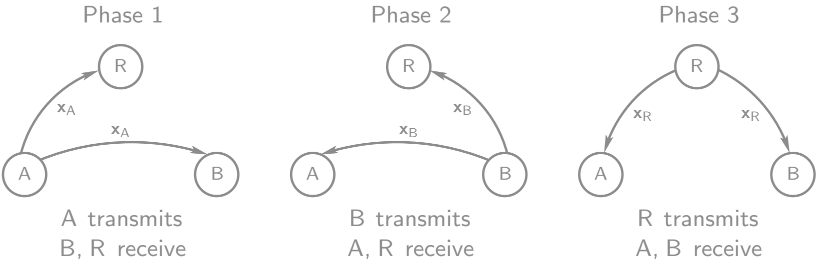
  - easy to implement in practice
  - network coding increases spectral efficiency



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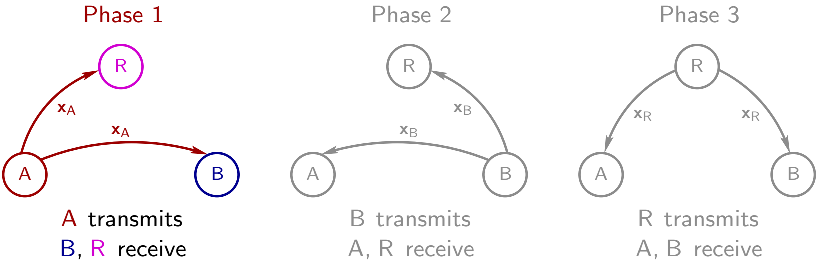

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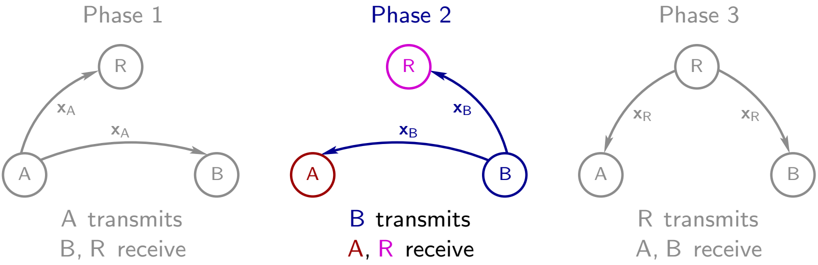

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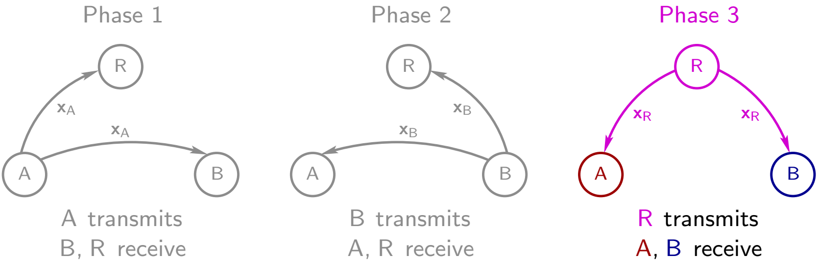

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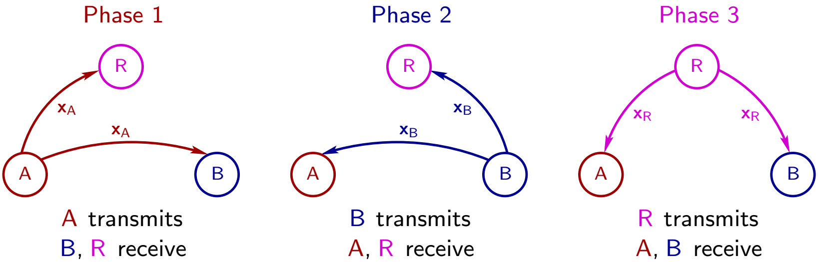

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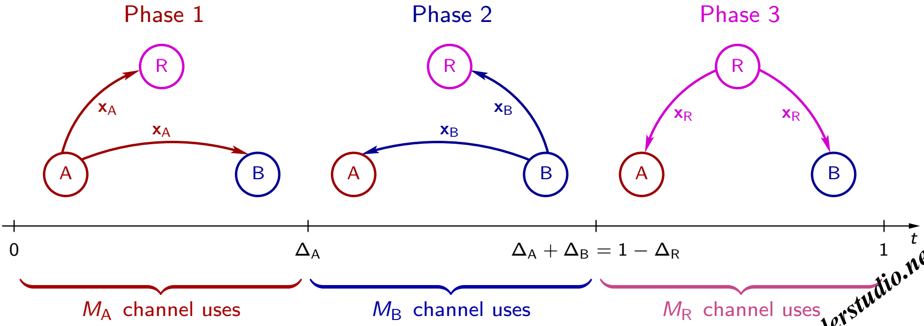
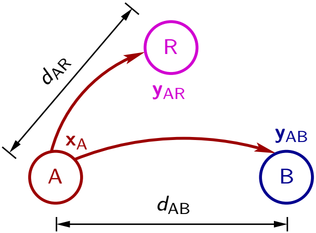
Notation

- M_i channel uses per node, $\sum_i M_i = M$, $i \in \{A, B, R\}$
- Transmit time $\Delta_i = M_i/M$ per node
- User A (B) transmits K_A (K_B) bits of information
- Rates $R_A = K_A/M_A, R_B = K_B/M_B$



Channel Model

- Assume **Gaussian channels**: $\mathbf{y}_{ij} = d_{ij}^{-n/2}\mathbf{x}_i + \mathbf{w}_{ij}$



- Assumptions
  - power constraint: $\mathrm{E}\left\{\|\mathbf{x}_i\|_2^2\right\}/M_i = 1$
  - noise statistics: $\mathbf{w}_{ij} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$
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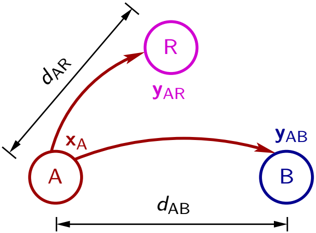

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Performance Limits

- **Information-theoretic limits** [KimMitTar, TrIT08]
  - outer bound of capacity region
  - achievable rates for DF-based transmission
- Compute sum-rate **optimal transmit time allocation**

$$\{\Delta_A^\star, \Delta_B^\star, \Delta_R^\star\} = \underset{\{\Delta_A, \Delta_B, \Delta_R\}}{\arg\max} \; R_S$$

with $R_S \triangleq R_A + R_B$, based on the outer bound

$$R_A \leq \min\{\Delta_A C(\gamma_{AR} + \gamma_{AB}), \Delta_A C(\gamma_{AB}) + \Delta_R C(\gamma_{RB})\}$$
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Performance Limits: Symmetric TWRC

- Restrict to the **symmetric TWRC** with $d_{\mathrm{AR}} = d_{\mathrm{BR}} = d_{\mathrm{AB}}/2$
  - $\gamma_{\mathrm{AR}} = \gamma_{\mathrm{BR}}$, $\gamma_{ij} = \gamma_{ji}$,
  - $[\gamma_{\mathrm{AR}}]_{\mathrm{dB}} = [\gamma_{\mathrm{BR}}]_{\mathrm{dB}} = [\gamma_{\mathrm{AB}}]_{\mathrm{dB}} + 10.6\,\mathrm{dB}$
  - $R \triangleq R_{\mathrm{A}} = R_{\mathrm{B}}$
  - $\Delta \triangleq \Delta_{\mathrm{A}} = \Delta_{\mathrm{B}}$, $\Delta_{\mathrm{R}} = 1 - 2\Delta$

- Here the **outer bound is tight for DF achievable rates**

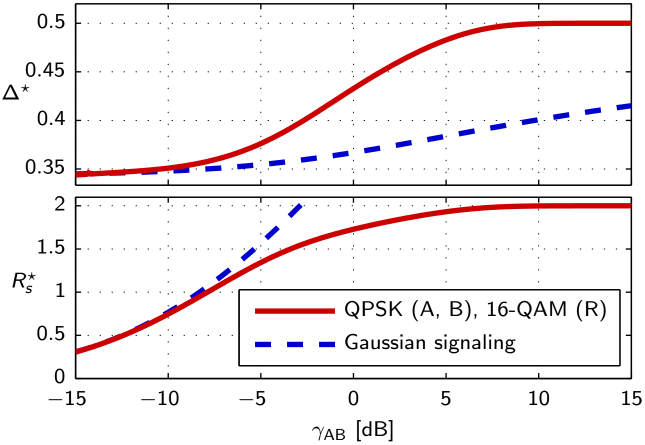
- Solving $\Delta^\star = \arg\max_\Delta R$ yields

$$\Delta^\star = \frac{C_{\mathrm{RB}}(\gamma_{\mathrm{RB}})}{C_{\mathrm{AR}}(\gamma_{\mathrm{AR}}) - C_{\mathrm{AB}}(\gamma_{\mathrm{AB}}) + 2C_{\mathrm{RB}}(\gamma_{\mathrm{RB}})}$$

if $C_{\mathrm{AB}}(\gamma_{\mathrm{AB}}) < \min\{2C_{\mathrm{RB}}(\gamma_{\mathrm{RB}}), C_{\mathrm{AR}}(\gamma_{\mathrm{AR}})\}$

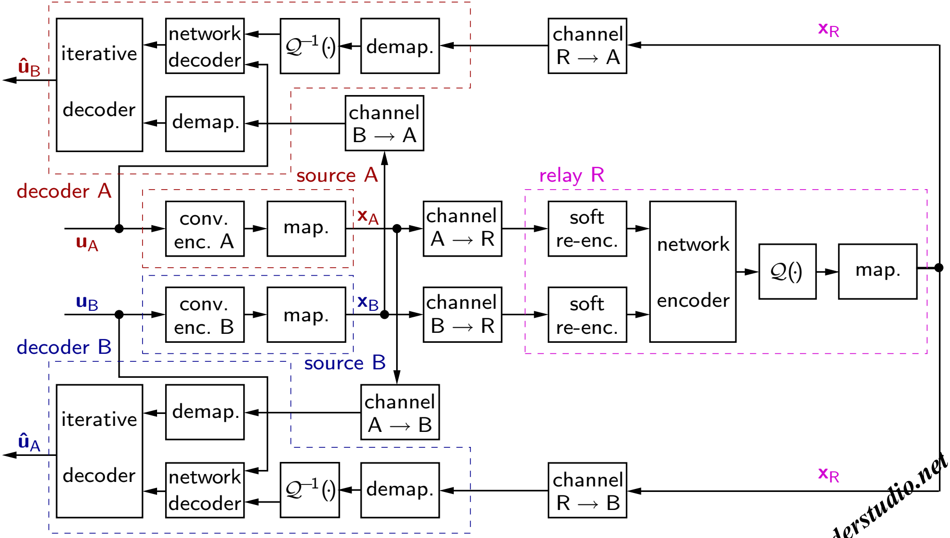
Performance Limits: Numerical Evaluation

- Example: QPSK map. at A, B; 16 QAM map. at R



- $\gamma_{AB} \to \infty \Rightarrow \Delta^\star \to 1/2$ and $R_S \to 2$

System Overview



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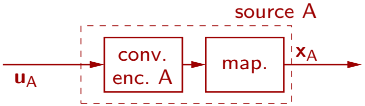

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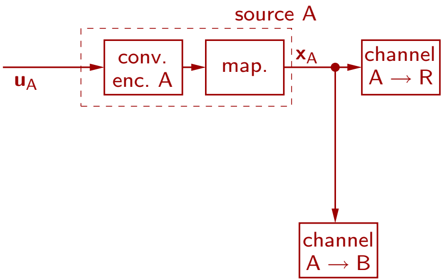

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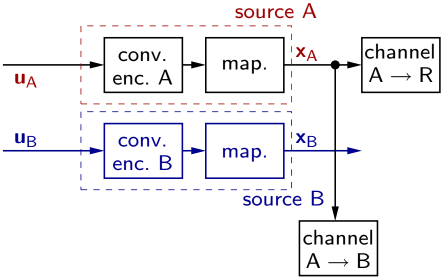

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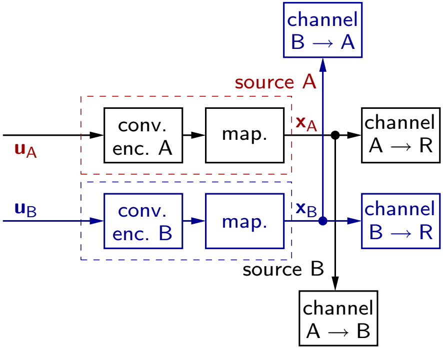

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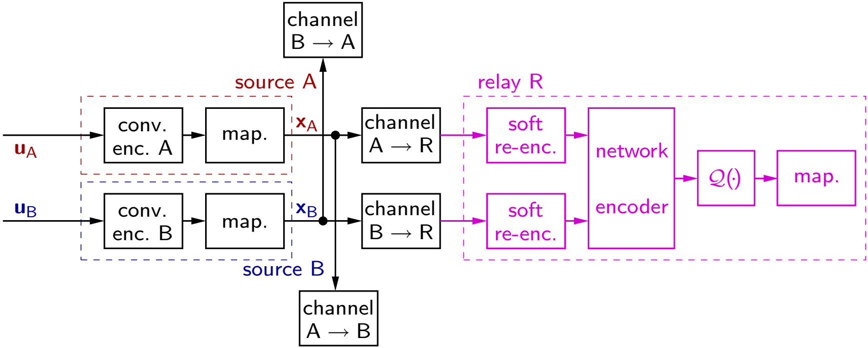

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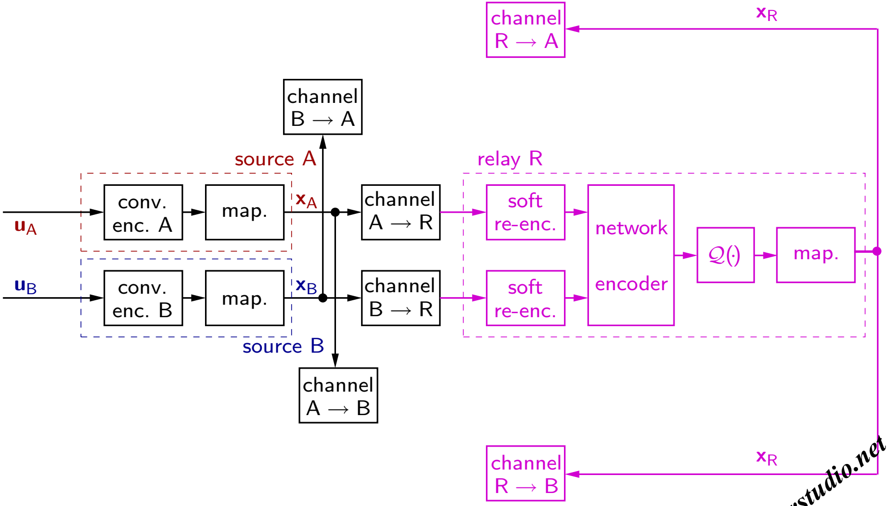

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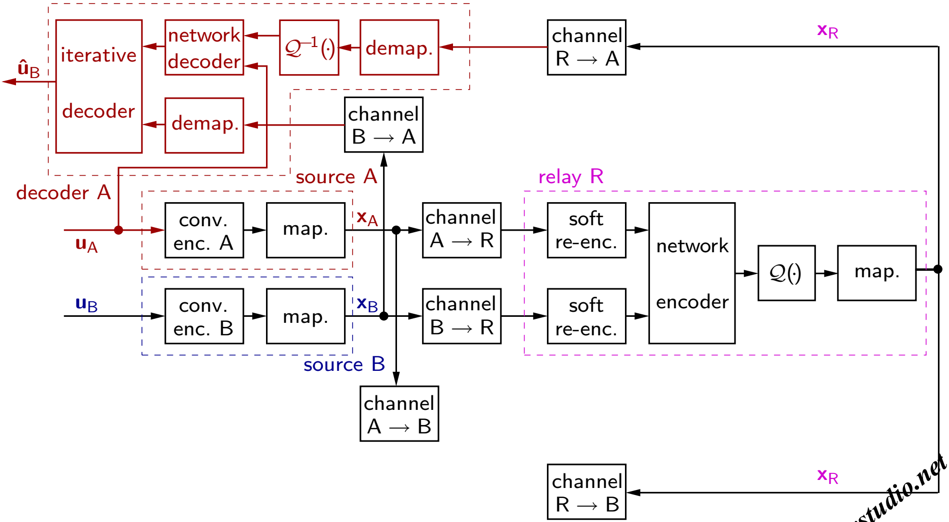

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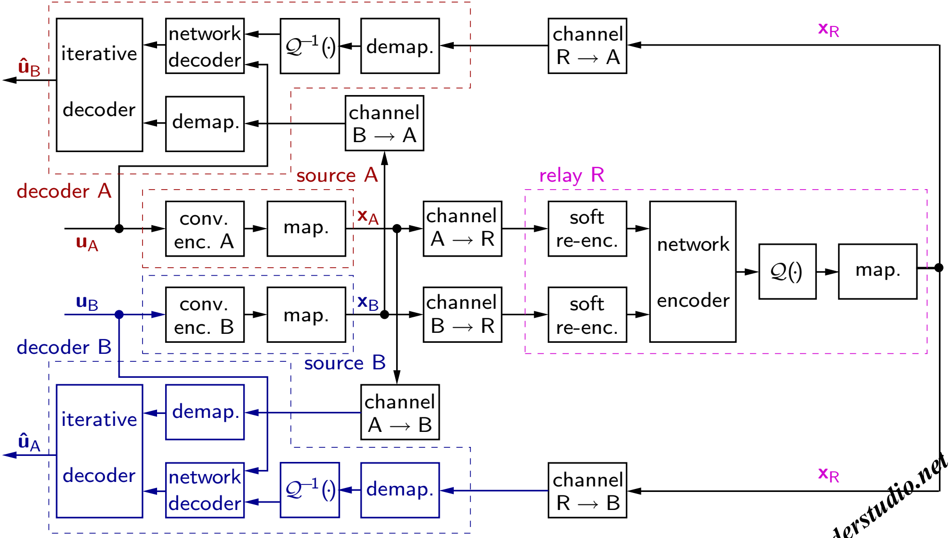
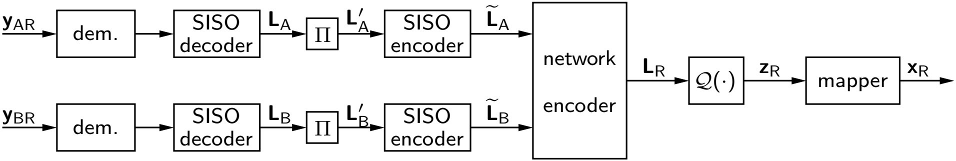


Relay Operation



- Demap and decode received signal
- Interleave and re-encode the decoder output
- Compute network-coded L-values $\mathbf{L}_R = \widetilde{\mathbf{L}}_A \boxplus \widetilde{\mathbf{L}}_B$
- Quantize $\mathbf{L}_R$ for digital transmission
- Map quantizer output to 16 QAM symbols

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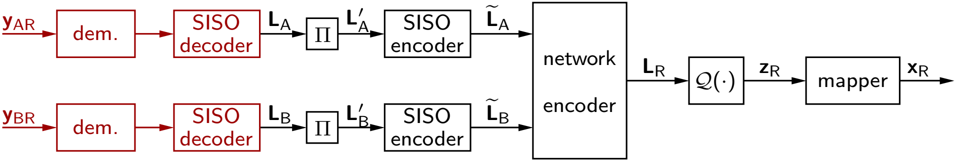

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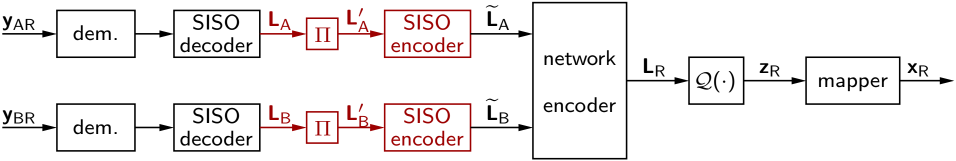

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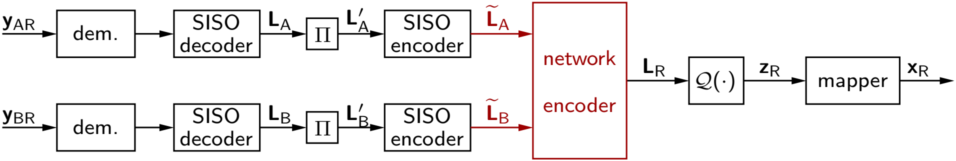

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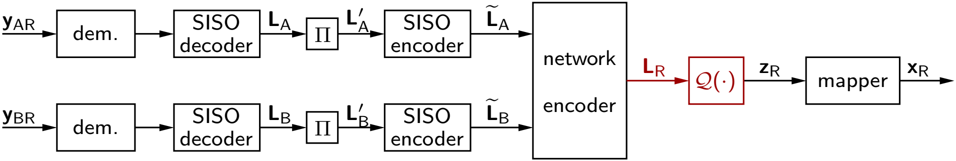

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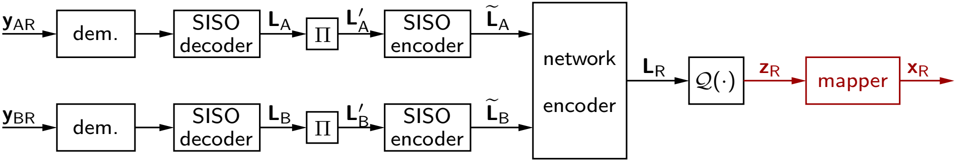

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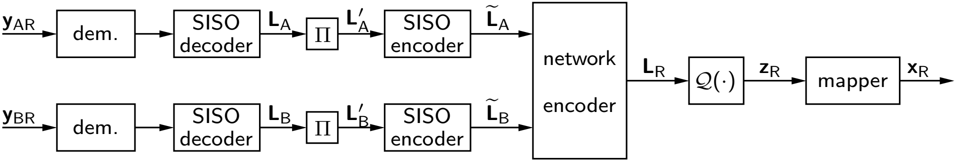
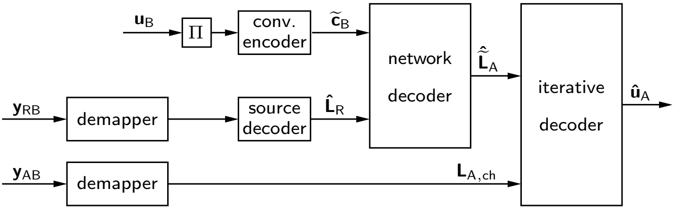


- Remarks
  - **Available information is used even if decoding fails**
  - **Design of quantizer and signal constellation is critical**
  - "Hard" processing possible if relay decoded both signals

Decoder Operation (1)

Decoder at node B:



- Demap signal received via direct link
- Reverse network coding performed at relay
  - source decoder computes an estimate of $\mathbf{L}_\mathrm{R}$
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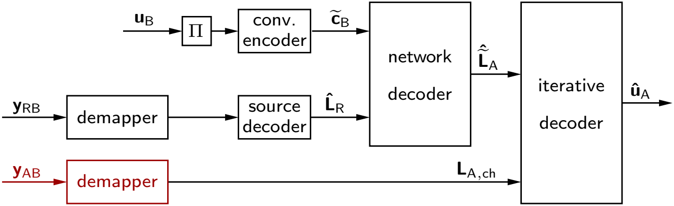

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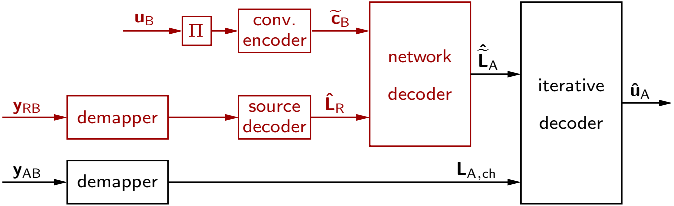

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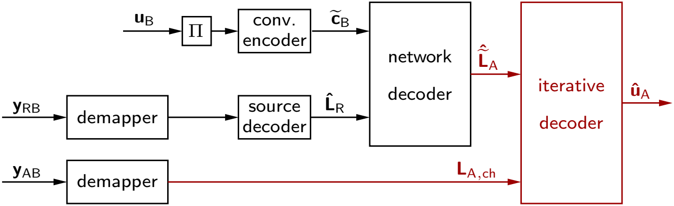

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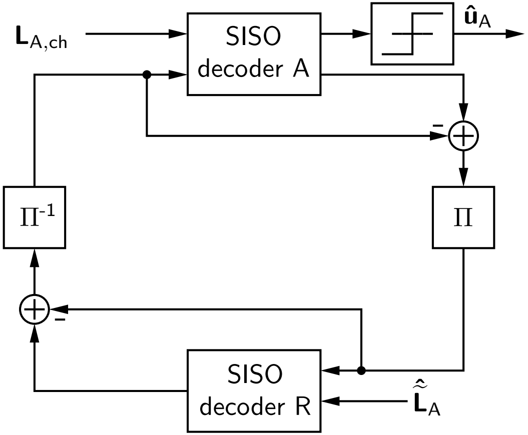
Iterative "turbo" decoder at node B:



Quantizer Design (1)

- Design a **scalar Q-level quantizer $\mathcal{Q}$** for L_R

$$\mathcal{Q}(L_R) = l_k \quad \text{if} \quad L_R \in \mathcal{I}_k, \qquad z_R = k \in \mathcal{Z} = \{0, 1, \ldots, Q-1\}$$

  - reproducer value l_k
  - quantization interval $\mathcal{I}_k$

- Quantization corresponds to **mapping** $p(z_R|l_R)$

- **Goal**: maximize mutual information $I(C_R; Z_R)$, $C_R = \widetilde{C}_A \oplus \widetilde{C}_B$

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Quantizer Design (2)

- One solution: **information bottleneck method** (IBM) [Tishby et al., Allerton99]
  - iterative algorithm finds **local optimum**
  - first applied to communications by Zeitler et al.
- Given $p^{\star}(z_R|l_R)$ **quantizer intervals $\mathcal{I}_k$ are fixed**
- fix l_k using L-value of **equivalent discrete channel** $p(z_{\mathrm{R}}|c_{\mathrm{R}})$

$$l_k = \log \frac{p(c_{\mathrm{R}} = 0 \,|\, z_{\mathrm{R}} = k)}{p(c_{\mathrm{R}} = 1 \,|\, z_{\mathrm{R}} = k)}, \quad k \in \mathcal{Z}$$

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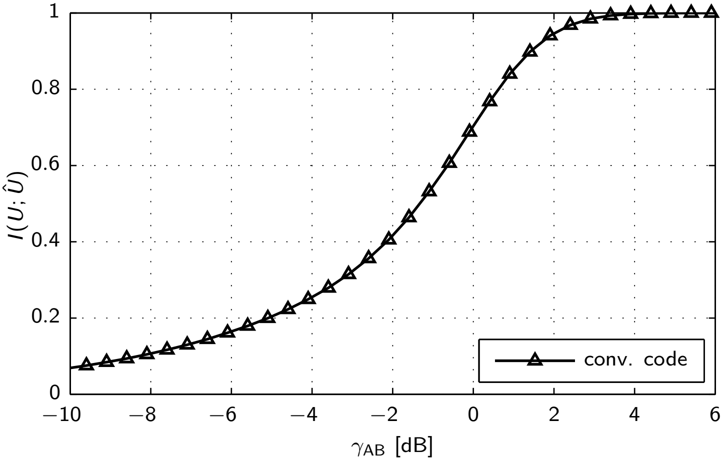
Signal Design

- Signal constellation and labels have to be designed carefully
- Here **Gray labeling is not optimal**
  - it matters which bit is in error!
- Use **binary switching** to optimize bit labels and constellation
  - **eliminates brute-force search**
  - $(2^4)! \approx 2 \cdot 10^{13}$ possibilities (16 QAM)
  - **quickly finds a "good" local optimum**
  - see [ZegerGersho, TrComm90] for details

Simulation Results: Information Rate

- $K_A = K_B = 256, R_S = 1, Q \in \{2, 4\}$, 1 decoder iteration



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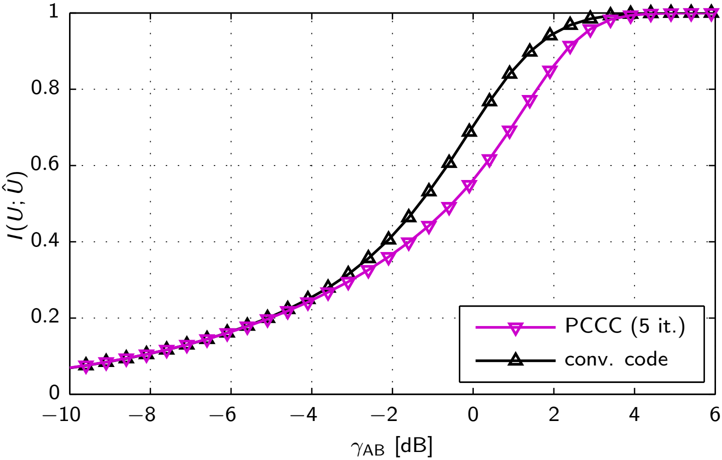

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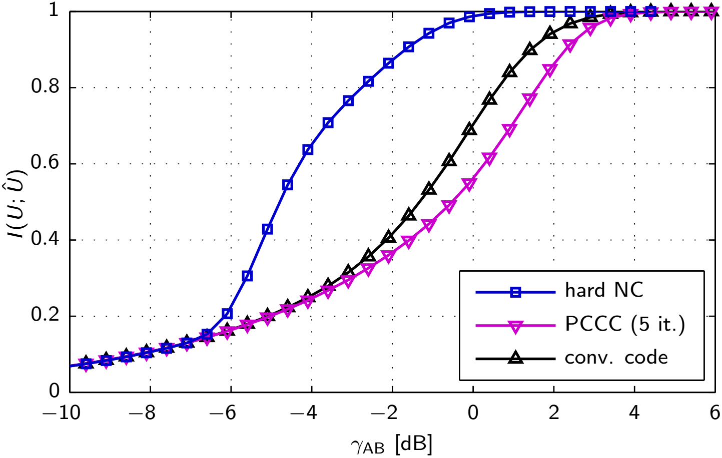

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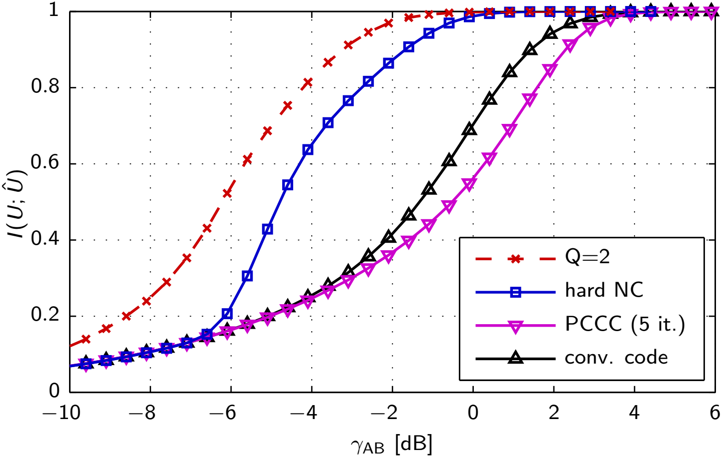

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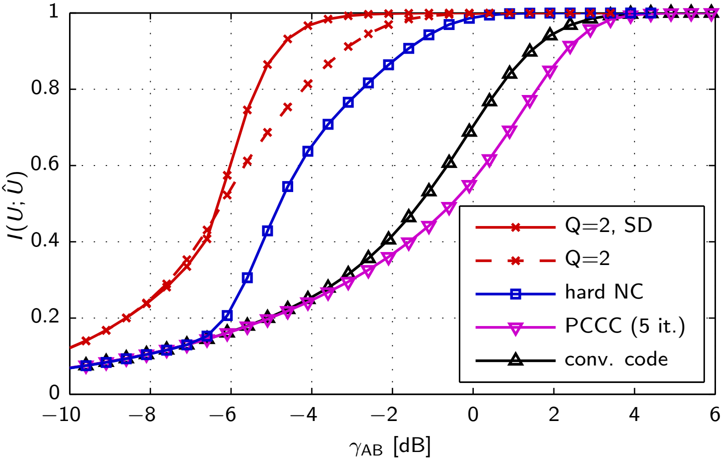

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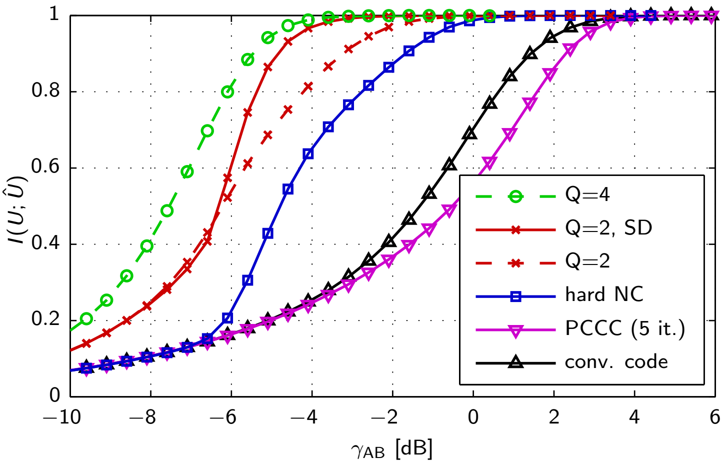

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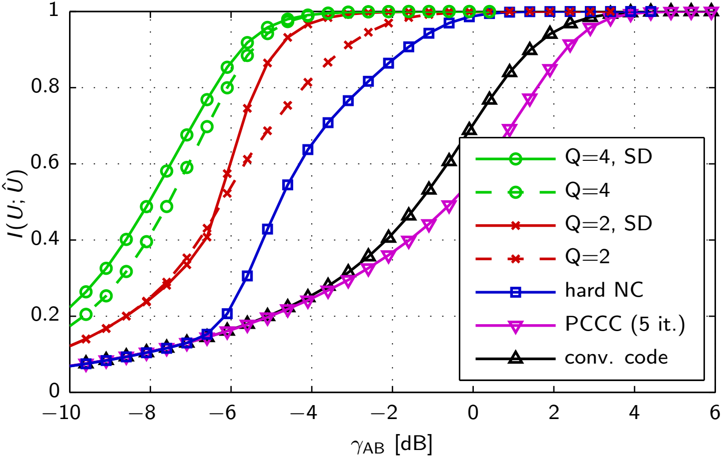

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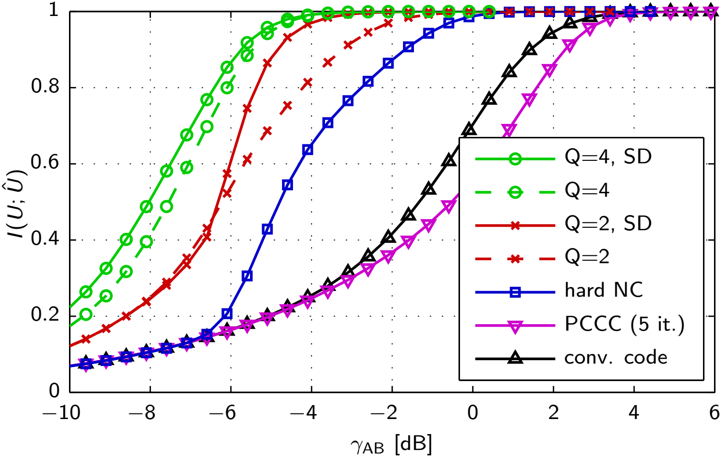
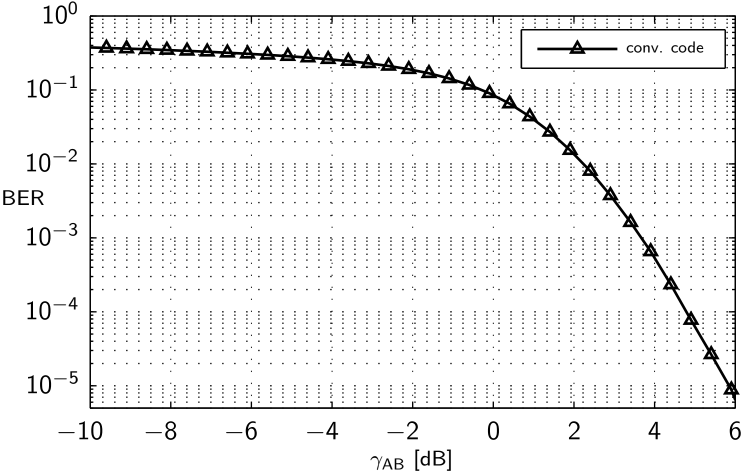


- **Gain of ∼ 3.5 dB over hard NC for a wide range of rates**

Simulation Results: Bit Error Ratio

- $K_\mathrm{A} = K_\mathrm{B} = 256, R_S = 1, Q \in \{2, 4\}$, 1 decoder iteration



Simulation Results: Bit Error Ratio

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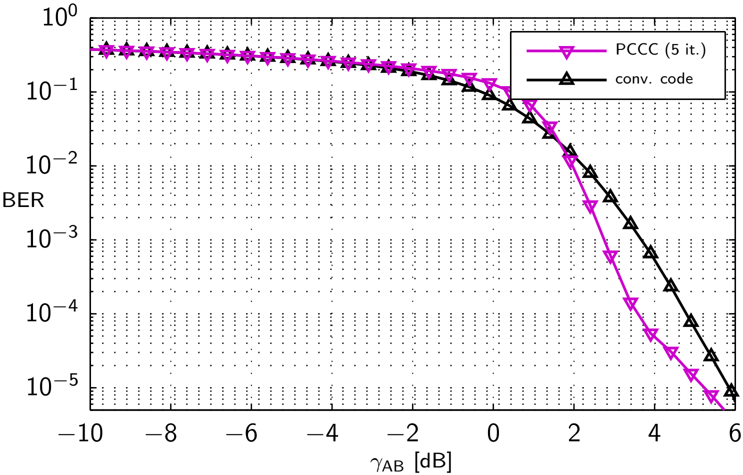

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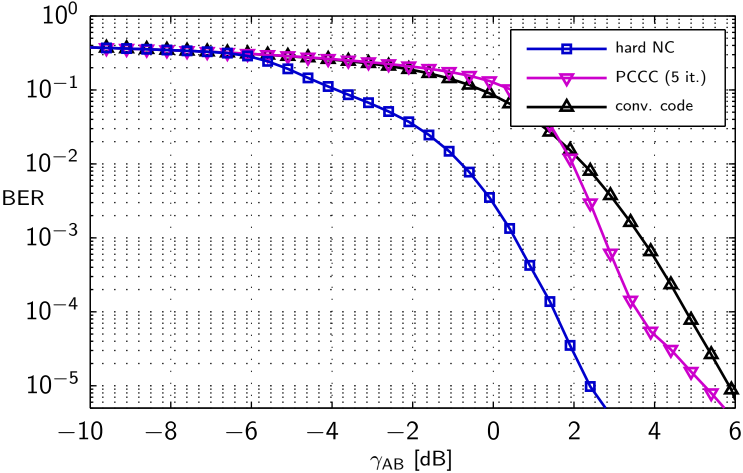

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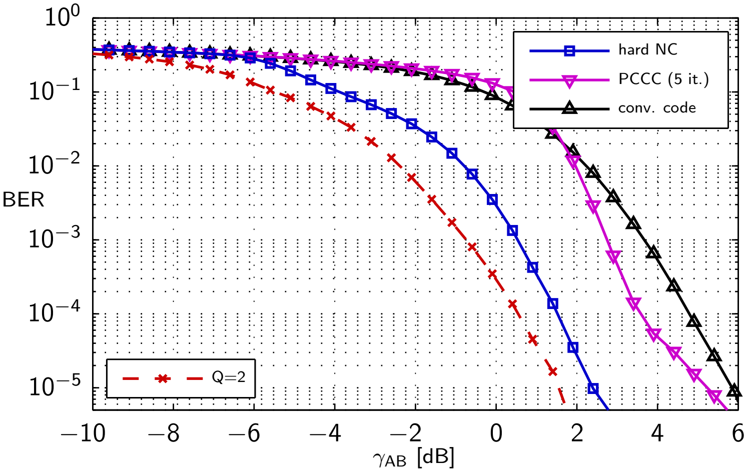

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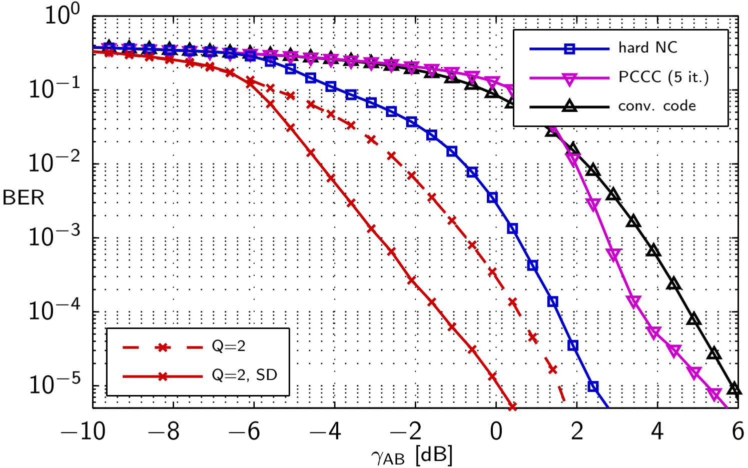

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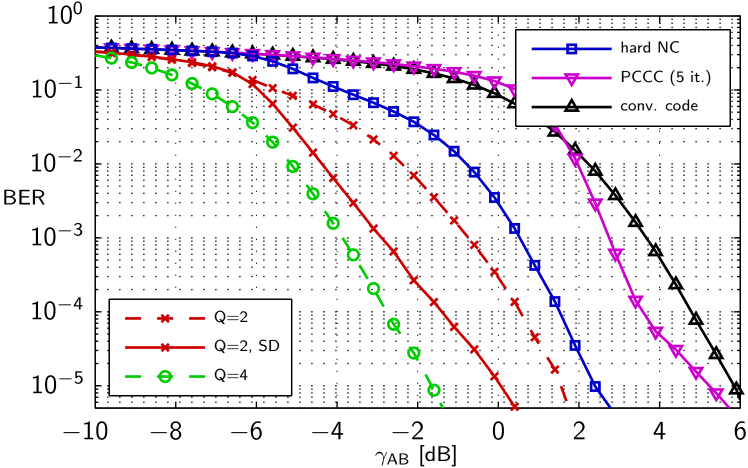

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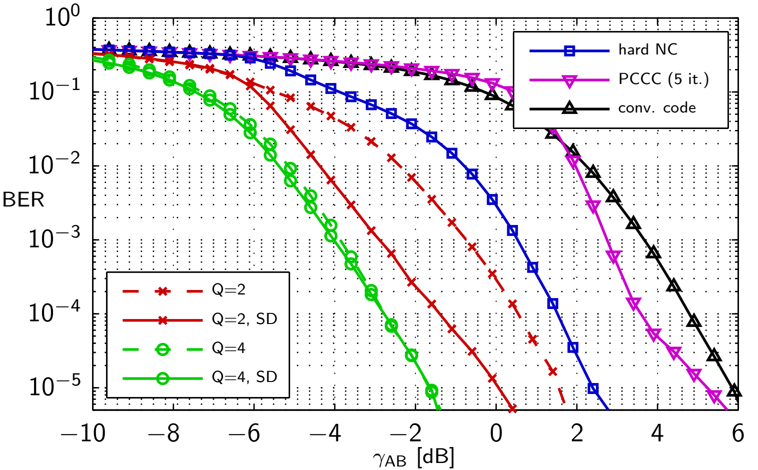

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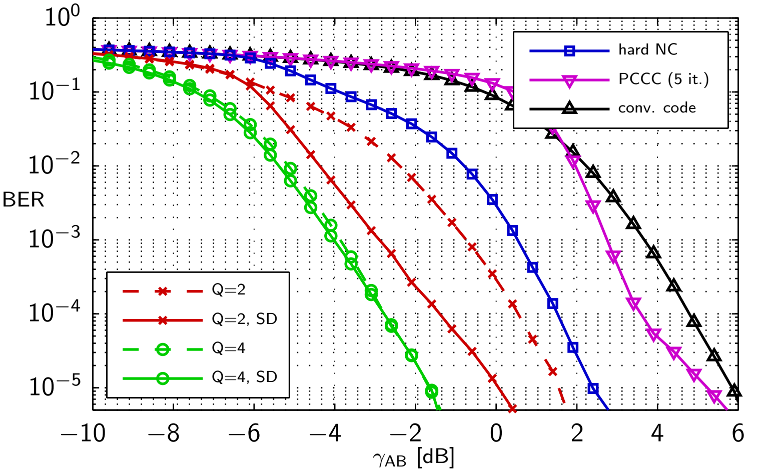
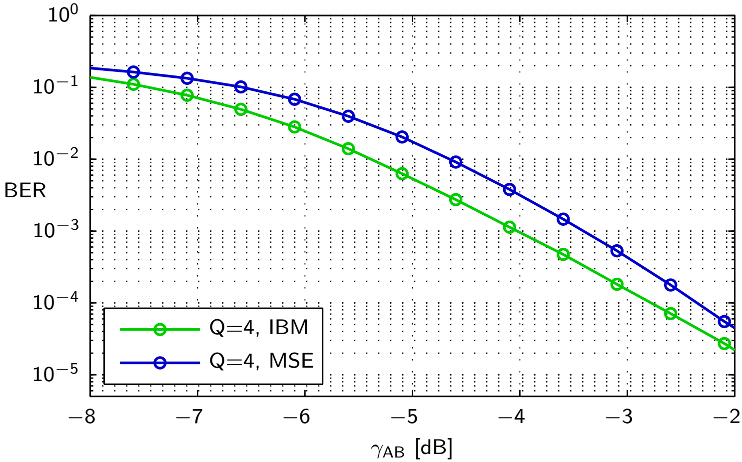


- **Gain of ∼ 4.5 dB over hard NC at BER** $= 10^{-3}$

Simulation Results: Quantizer Design

- IBM quantization vs. MSE-optimal quantization ($Q = 4$)



- **IBM design gains between 0.5 dB and 1 dB**

Conclusions and Outlook

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- Novel soft network coding scheme for the TWRC
- Iterative joint network-channel decoding
- Significant performance gains over “hard” network coding

Outlook:

- Extension to non-symmetric TWRC
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Thank you!