Modified Earliest Decoding for Random Network Codes

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Motivation

- From Computer & Electronic engineering perspective:
  - Network Coding offers a range of challenges regarding the practical implementation thereof.

Our aim

bridging the gap between:

- theoretical work – advantages
- practical implementation – challenges

In this paper:

- Aim to reduce use of network resources through:
  - Reducing decoding complexity
  - Reducing decoding delay
  - Speeding up decoding process

Related Work

- Gaussian Elimination in RLNC networks
- Decoding through evaluation of coding vectors

Decoding time

- k packets of size m
- Complexity: $O(m \cdot k^2)$
- Exponential growth with increase in packet size

Decoding delay

- Commencing only after all packets received

[Chou et.al. 2003]

Related Work

- Earliest Decoding
  - Gaussian Elimination on selection of packets when possible

$$\begin{pmatrix} a & & & & \\ a & b & & & \\ & b & & & e \\ a & & & d & \\ a & & c & d & e \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

  - Less resource intensive
  - Same complexity – smaller blocks
  - Smaller decoding delay

[Chou et.al. 2003]

Related Work

- Luby Transform Codes
  - Message passing algorithm for decoding
    - Sufficient decoding times
    - Low complexity
  - Not optimal when random encoded packets are received

[Luby 2002]

Modified Earliest Decoding

- Using idea of message passing-type decoding in conjunction with Earliest Decoding
  - Lower complexity than Gaussian Elimination
  - Lower decoding delay
  - Less resource intensive
- Implementable in RLNC networks:
  - Without need for specific encoding

RLNC: encoding

- Consider single source network
  - generates information vector $(x_1\ x_2\ \dots\ x_k) \in \mathbb{F}^k$
  - Transmit over sufficient cut network to receiver nodes $\{t_1, \dots, t_r\}$
  - Intermediate nodes implement RLNC forming encoded packets:

$$\boldsymbol{y}_j = \sum_{i=1}^{k} g_{ij} x_i$$

  - Global encoding vector $\boldsymbol{g}_j$ is included in message header: records linear combinations performed on each packet

Traditional avalanche type decoding

- Method:
  - Find vector $x \in g_j$ with Hamming weight $w_H(x) = 1$
  - XOR with other relevant coding vectors $w_H(x') = 1$
  - Obtaining new vector x' of weight
  - In decentralized network: small probability of obtaining randomly

[MacKay 2005]

Modified Earliest Decoding

- We look at:
  - Hamming distance between coding vectors
  - Higher probability to obtain randomly

Modified Earliest Decoding

- Method:
  - Find 2 packets $x, y \in g_j$ with Hamming distance $d_H(x, y) = 1$
  - Obtain new code vector $z = x \oplus y$ where $w_H(z) = 1$
  - If $w_H(x) < w_H(y)$
    - Replace y with z
    - Reinsert x into decoding buffer
  - Process repeats as long as $d_H(x, y) = 1$ is present

Example

- Collect encoding vectors
- Store in decoding buffer

$$g_\tau = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix}$$

Example

- Evaluate $d_H(x, y)$ of encoding vectors
- If $d_H(x, y) = 1$, calculate $z = x \oplus y$

$$
\begin{matrix} a \\ b \\ c \\ d \end{matrix}
\begin{pmatrix}
1 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 \\
0 & 1 & 0 & 1
\end{pmatrix}
$$

$d_H(a,b)=1$
$z = [0\ 0\ 0\ 1]$

Example

- If $w_H(x) < w_H(y)$
  - Reinsert x into decoding buffer
  - z replaces vector y

a	0	0	0	1	$w_H(a)=3$
b	1	1	0	0	$w_H(b)=2$
c	0	1	1	1	
d	0	1	0	1	

$z = [0\ 0\ 0\ 1]$

Example

- Repeat steps as long as there exist

$$d_H(x, y) = 1$$

$$\begin{array}{c} a' \\ b \\ c \\ d \end{array} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{array}{l} \\ \\ w_H(c)=3 \\ w_H(d)=2 \end{array}$$

$d_H(c,d)=1$

$z = [0\ 0\ 1\ 0]$

Example

- Repeat steps as long as there exist

$$d_H(x, y) = 1$$

a'	0	0	0	1	$w_H(a')=1$
b	1	1	0	0	
c'	0	0	1	0	
d	0	1	0	1	$w_H(d)=2$

$d_H(a', d)=1$

$z = [0\ 1\ 0\ 0]$

Example

- Repeat steps as long as there exist

$$d_H(x, y) = 1$$

a'	0	0	0	1	
b	1	1	0	0	$w_H(b)=2$
c'	0	0	1	0	
d'	0	1	0	0	$w_H(d')=1$

$d_H(b,d')=1$

$z = [0\ 1\ 0\ 0]$

Example

- Identity matrix is present in decoding buffer

$$\begin{matrix} a' \\ b' \\ c' \\ d' \end{matrix} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Evaluation

- Decoding complexity
- Decoding performance

Decoding complexity

- Calculation of Hamming distance of packets: $\mathcal{O}(k^2)$
- Iterative process – completed in $\mathcal{O}(\log k)$

Decoding performance

- Network implementing RLNC:
  - Intermediate nodes:
    - Packets encoded randomly, independently, non-zero
    - Global encoding vector in header
  - Receiver nodes:
    - Assume Gaussian distribution of k source packets
    - All-zero vector is present in decoding buffer
    - Obtains new packet at every transmission instance λ
    - Evaluate encoding vectors of packets
    - Calculate Hamming distances

[Hessler et. al. 2010]

Decoding performance

- Decoding can commence when 2 packets have encoding vectors with Hamming distance of 1.
- Probability of commencing decoding at $\lambda=1$
  - Packet containing single source packet

$$\rho = \frac{k}{2^k - 1}$$

Decoding performance

- Probability of commencing decoding at $\lambda=2$
  - Either packet containing single source packet
  - Packets' coding vectors have $d_H(x,y) = 1$
  - $\rho_{d_H=1} = \frac{1}{C}\sum_{j=2}^{k-1}(k-j)\binom{k}{j}$, where $C = \binom{2^k - 1}{2}$

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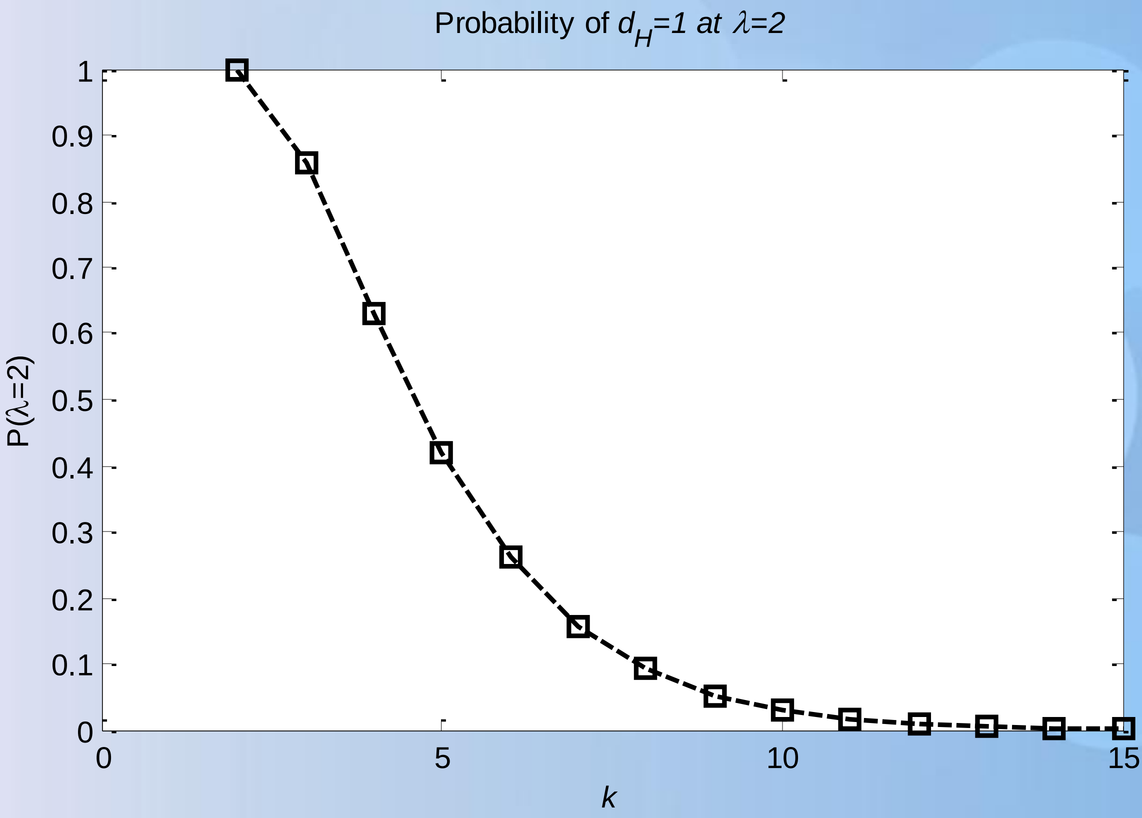
Decoding performance

- Probability of commencing decoding at $\lambda=2$

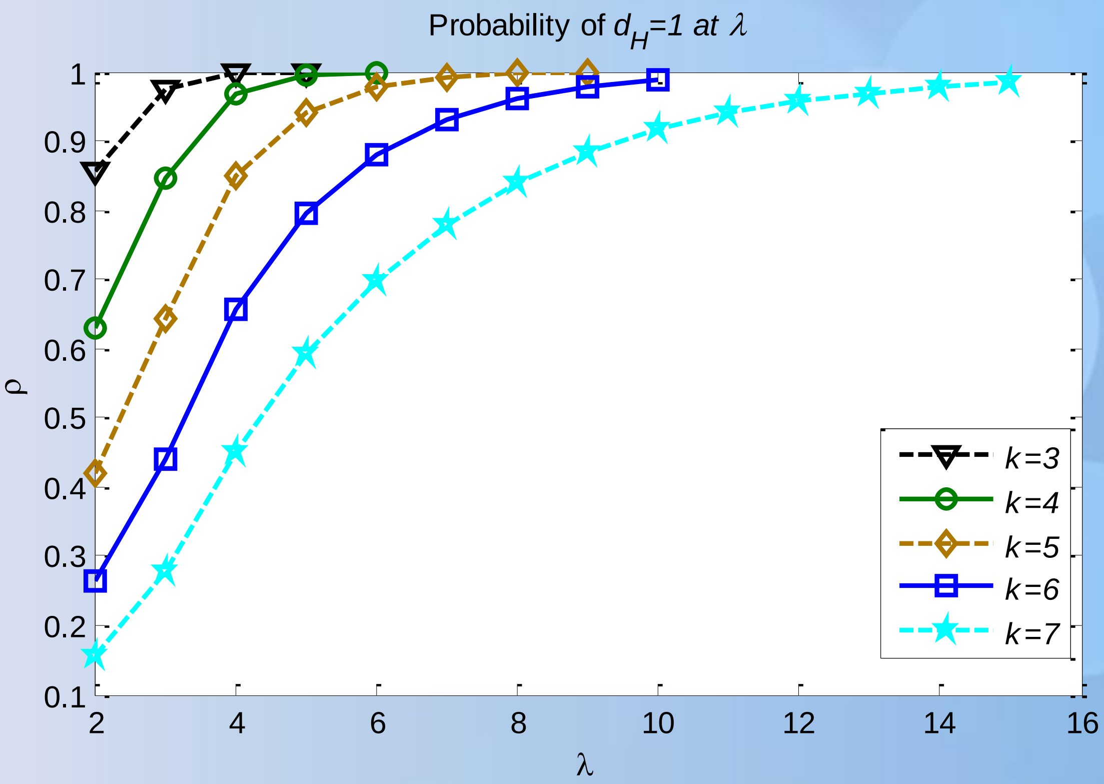


Decoding performance

- Probability of commencing decoding at $\lambda > 2$
  - A packet containing single source packet
  - Any 2 received packets' coding vectors have $d_H(x, y) = 1$
  - Probability can be approximated by:
  - $\rho_\lambda \cong 1 - (1 - \rho_{\lambda=2})^D$, where $D = \frac{\lambda(\lambda - 1)}{\lambda/_2 + 1} - \frac{1}{2}$

Decoding performance

- Probability of commencing decoding at $\lambda > 2$ for different values of k.



Conclusion

- Modified Earliest Decoding
  - Decoding is less resource intensive
  - Decoding time: smaller decoding complexity than GE
  - Decoding delay: Probability commencing decoding at $\lambda<k$ is high
- No need to modify coding technique at nodes
- Complexity is restricted to decoder only

Future work

- These are preliminary results:
  - Need to calculate requirements for successful decoding with this method
  - Need to determine when it is more effective to use GE that wait for additional packets for MED
- Determine optimal parameters for different limitations:
  - Decoding resources
  - Decoding Time

Thank You