Research Landscape of Physical-Layer Network Coding

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Slides in this talk based on
“Physical-layer Network Coding: Tutorial, Survey, and Beyond”. A 60-page monograph.
(http://arxiv.org/abs/1105.4261)

Background

- First proposed in a challenge paper of Mobicom 06 [1]
- A subfield of Network Coding with some momentum in last two years
  - Special journal issues dedicated to the topic.
  - Citations to-date: 468 (Google Scholar)
- Fundamental impact on *Communications*, *Information Theory*, and *Networking Research*.

[1] S. Zhang, S. C. Liew, P. P. Lam, "Physical-Layer Network Coding," *ACM Mobicom 2006* .

Similar idea in
[2] P. Popovski and H. Yomo, "The anti-packets can increase the achievable throughput of a wireless multi-hop network," *ICC 2006*.

Outline

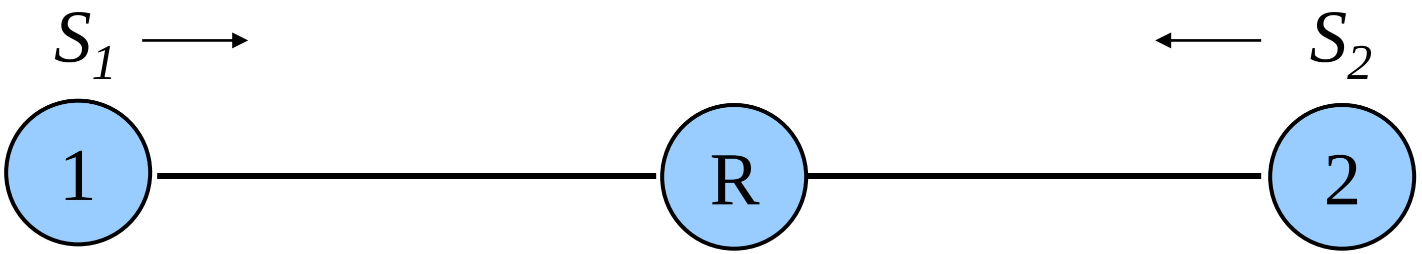
- Basic idea of PNC
- Communication-theoretic research
- Information-theoretic research
- Network-theoretic research
- Conclusion

What is PNC about?

- Traditional view in wireless networking: interference is bad.
- PNC turns things around by exploiting *network coding performed by nature*.

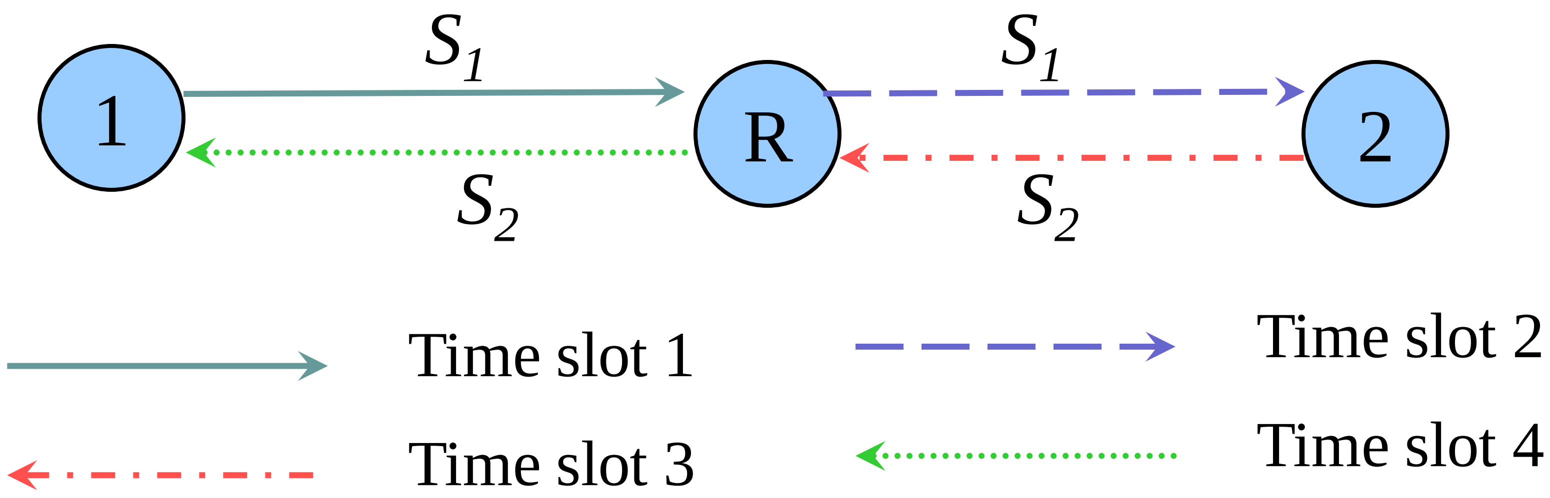
  NC = Nature's Coding?
- When electromagnetic waves are superimposed, they add, a form of NC.
- Benefit of PNC: (i) boosted throughput; (ii) reduced processing.

Simplest Set-up: Two-Hop Linear Network



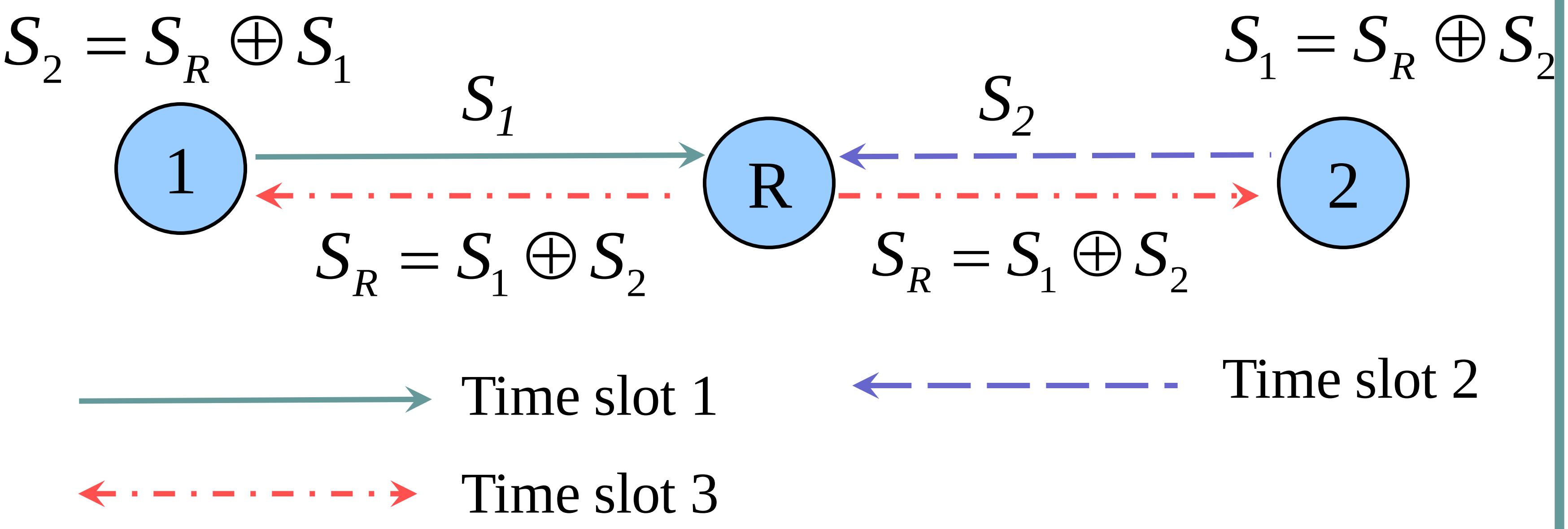
- Two-way Relay Channel (TWRC)
  - No direct channel between nodes 1 and 2.
  - Half duplex: nodes cannot transmit and receive at the same time.
  - What is the minimum number of time slots needed for nodes 1 and 2 to exchange one packet via relay node *R*?

Traditional Scheduling (TS)



Transmissions non-overlapping in time

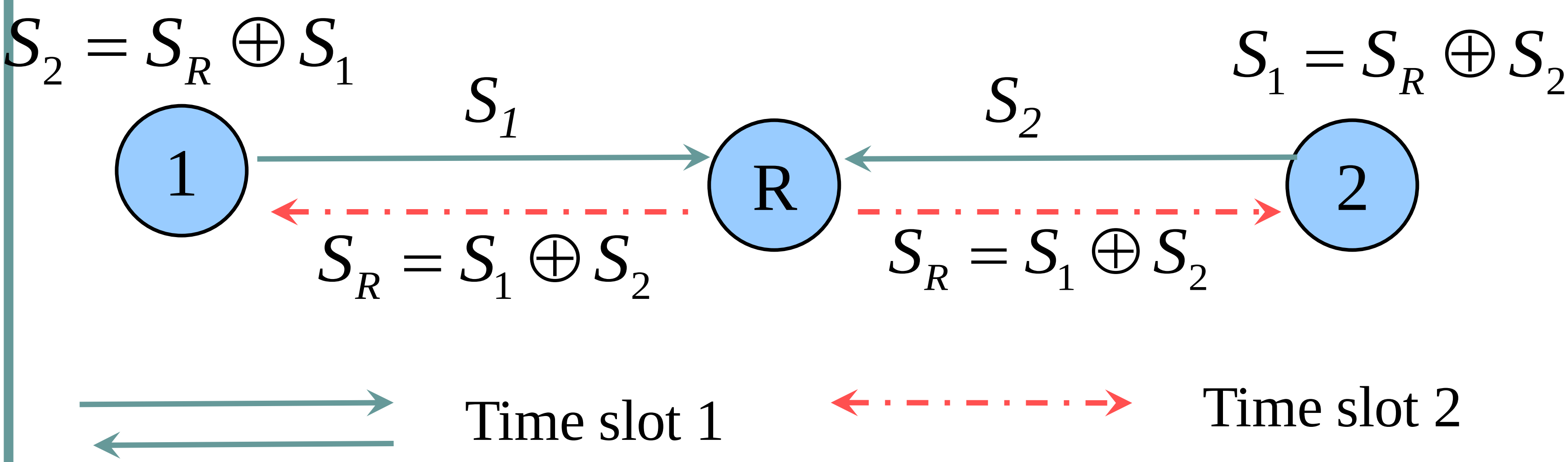
Straightforward Network Coding (SNC)



Node R uses one time slot to broadcast

Transmissions by nodes 1 and 2 still non-overlapping

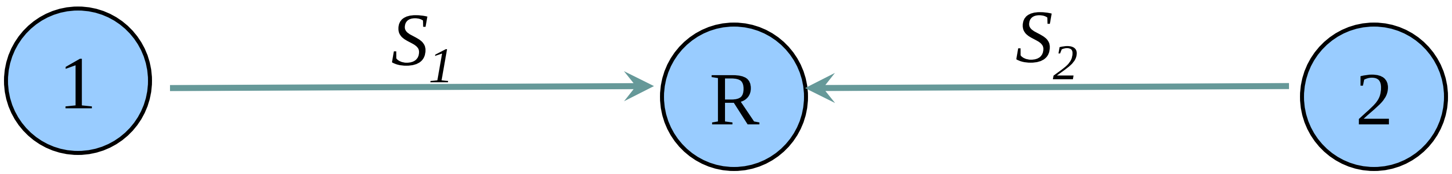
PNC



Transmissions by nodes 1 and 2 are simultaneous!

PNC Modulation Mapping (1)

- Assume BPSK modulation



$$r_R(t) =$$

$$\begin{aligned} & s_1(t) && + && s_2(t) \\ = \quad & a_1 \cos(\omega t) && + && a_2 \cos(\omega t) \\ = \quad & && (a_1 + a_2)\cos(\omega t) && \end{aligned}$$

$$\text{bit } = 1 \Leftrightarrow a_i = -1$$

$$\text{bit } = 0 \Leftrightarrow a_i = 1$$

$$a_1 \oplus a_2 \equiv a_1 a_2$$

PNC Modulation Mapping (2)

a_1	a_2	$a_1 + a_2$	$a_R = a_1 a_2$
1	1	2	1
-1	1	0	-1
1	-1	0	-1
-1	-1	-2	1

Reception at relay (column $a_1 + a_2$) Transmission by relay (column $a_R = a_1 a_2$)

Outline

- Basic idea of PNC
- Communication-theoretic research
  - Most work focuses on 3-node chain (TWRC)
- Information-theoretic research
- Network-theoretic research
- Conclusion

Communications-Theoretic Research

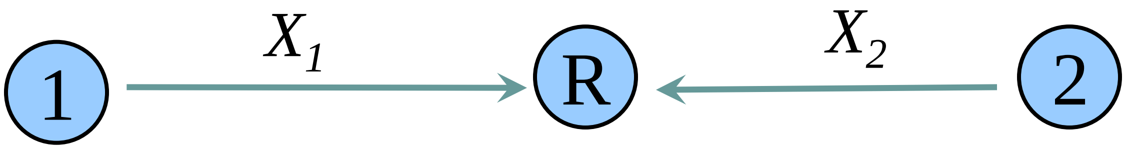
Focuses of existing papers in literature:

- Other variants on of PNC:
  - e.g., Analog Network Coding (ANC)[3]
- Implementation challenges of PNC:
  - Synchronization: phase, frequency, and symbol timing.
- Channel-coded PNC:
  - End-to-end versus link-by-link

This talk focuses on synchronization issue and link-by-link channel-coded PNC

[3] S. Katti, S. Gollakota, D. Katabi, "Embracing Wireless Interference: Analog Network Coding," ACM SigComm 2007.

Synchronization Penalties
(unchannel-coded case) (1)



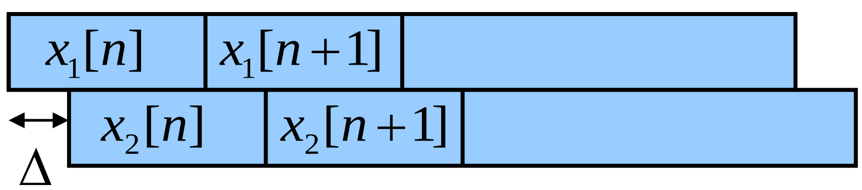
$$y_R(t) = \sum_n x_1[n] h_{1R} p(t - nT) + \sum_n x_2[n] h_{2R} p(t - \Delta T - nT) + w_R(t)$$

$y_R(t) =$ received baseband signal

$p(t) =$ pulse shape

$\Delta =$ symbol offset

$w_R(t) =$ noise



Synchronization Penalties
(unchannel-coded case) (2)

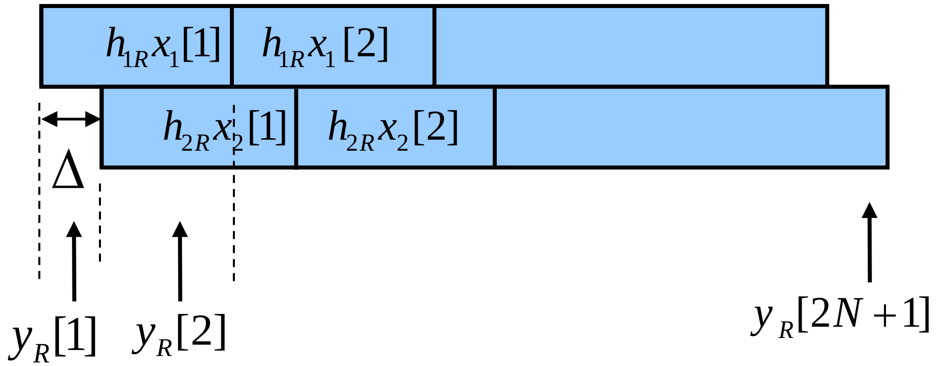
- Assumption:
  - Transmitters do not perform precoding to synchronize
  - Relay knows the channels h_{1R}, h_{2R} and symbol offset Δ
- Suboptimal results in [1] [7]
- Better results in [8]

[7] S. Zhang, S. C. Liew, P. P. Lam, "Physical-Layer Network Coding," http://arxiv.org/abs/0704.2475, April 2007
"On the Synchronization of Physical-Layer Network Coding," *IEEE Information Theory Workshop,* Oct 2006.
[8] L. Lu, S. C. Liew, S. Zhang, "Decoding Algorithm for Asynchronous Physical-Layer Network Coding," *IEEE ICC 2011*.

Synchronization Penalties (unchannel-coded case) (5)

Recent work [8] assumes oversampling and uses Belief Propagation (BP) to compute

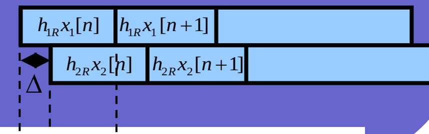
$$P(x_1[n] \oplus x_2[n] \mid y_R[1], \ldots, y_R[2N+1]) \text{ for } n = 1, \ldots, N$$



Two samples per symbol

$$P(x_1[n] \oplus x_2[n] = x \mid y_R[1], \ldots, y_R[2N+1]) = \sum_{x_1[n] \oplus x_2[n] = x} P(x_1[n], x_2[n] \mid y_R[1], \ldots, y_R[2N+1])$$

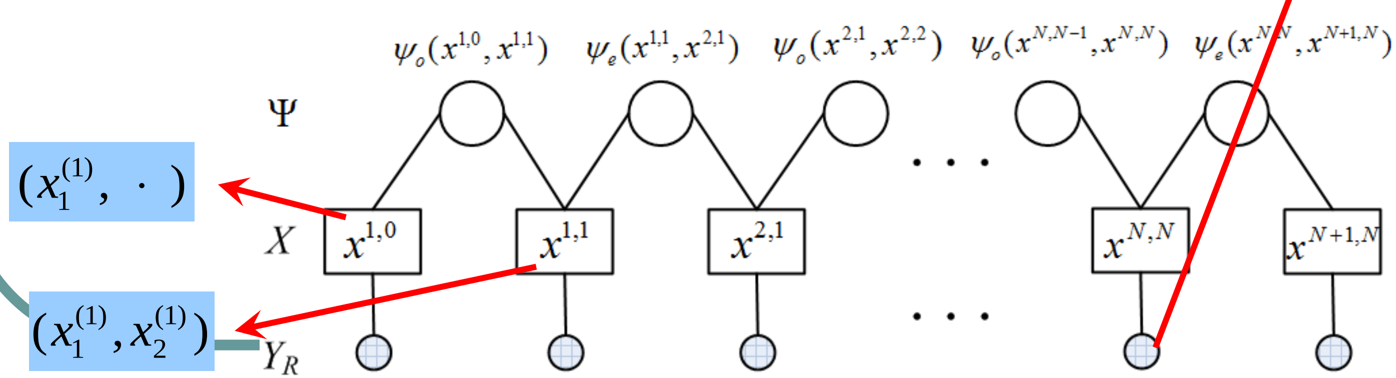
Synchronization Penalties
(unchannel-coded case) (6)



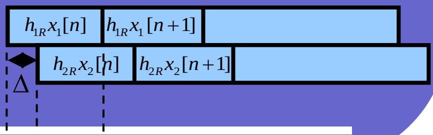
- Tanner Graph for BP decoding of $P(x_1[n], x_2[n] \mid y_R[1], \ldots, y_R[2N+1])$
  - Yields exact ML decoding because of the tree structure

$$\begin{pmatrix} \Pr\left(x_1^{(N)}=0, x_2^{(N)}=0 \mid y_R^{(N,N)}\right) \\ \Pr\left(x_1^{(N)}=0, x_2^{(N)}=1 \mid y_R^{(N,N)}\right) \\ \Pr\left(x_1^{(N)}=1, x_2^{(N)}=0 \mid y_R^{(N,N)}\right) \\ \Pr\left(x_1^{(N)}=1, x_2^{(N)}=1 \mid y_R^{(N,N)}\right) \end{pmatrix}$$

where $y_R^{(N,N)} = (1-\Delta)[h_{1R}x_1^{(N)} + h_{2R}x_2^{(N)}] + w^{(N,N)}$

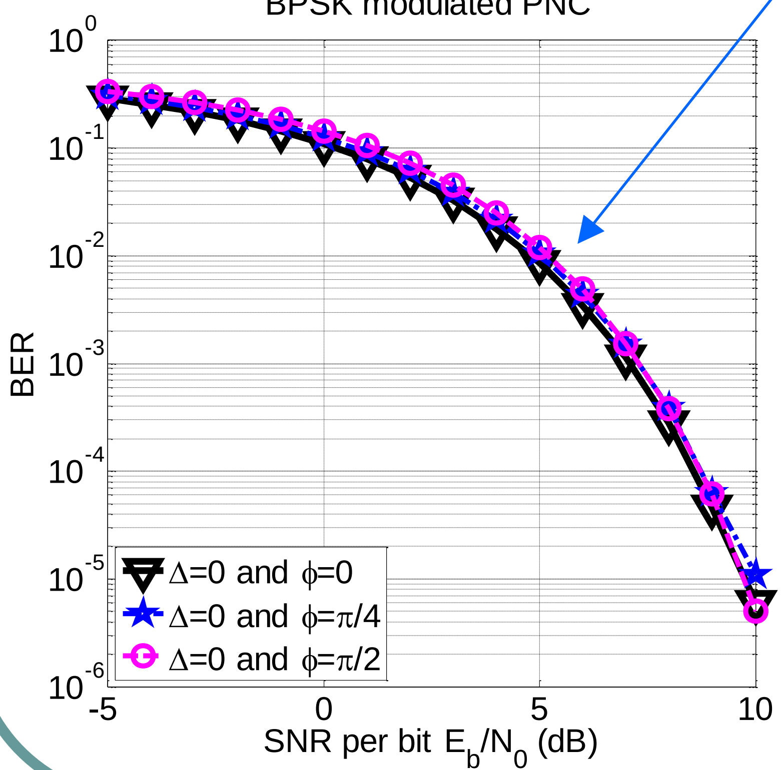


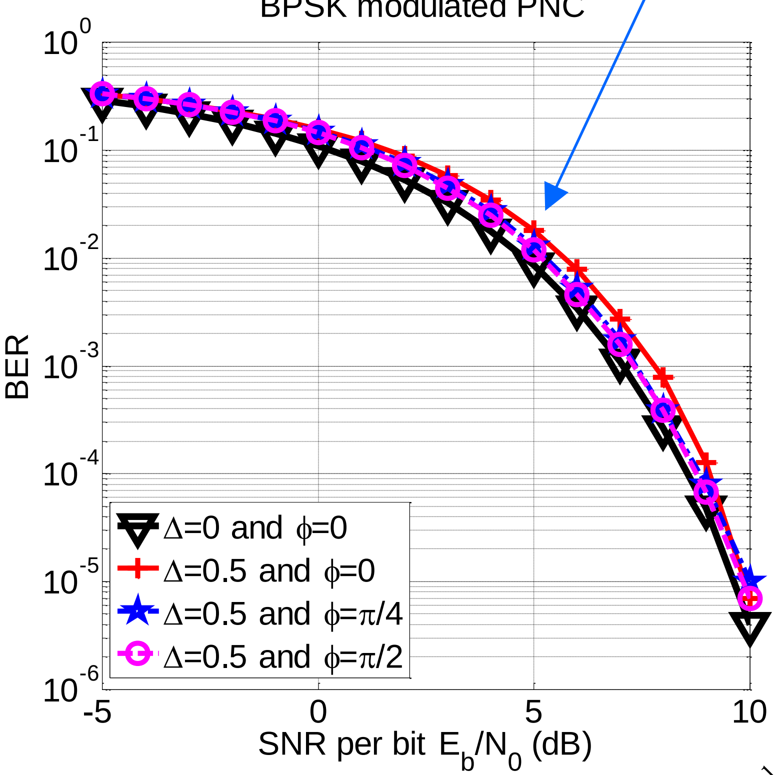
Synchronization Penalties
(Unchannel-coded case) (7)



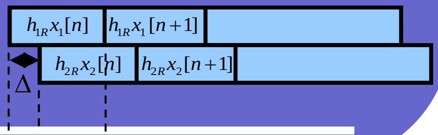
BPSK results from recent work [8]

Little penalty with phase and symbol asynchronies, thanks to BP decoding

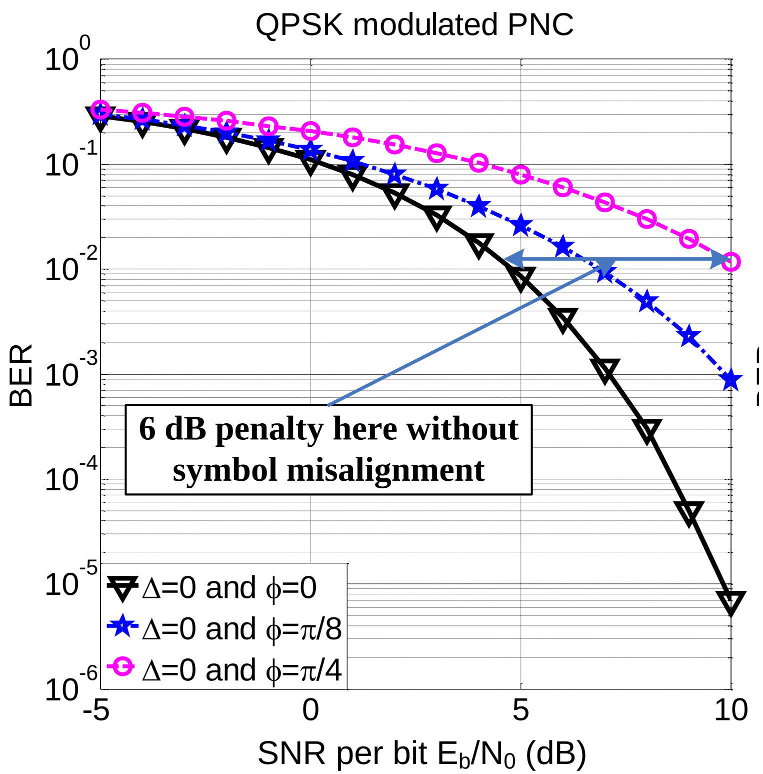




Synchronization Penalties
(Unchannel-coded case) (7)



QPSK results from recent work [8]

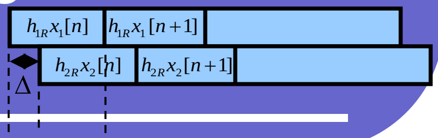


(a)

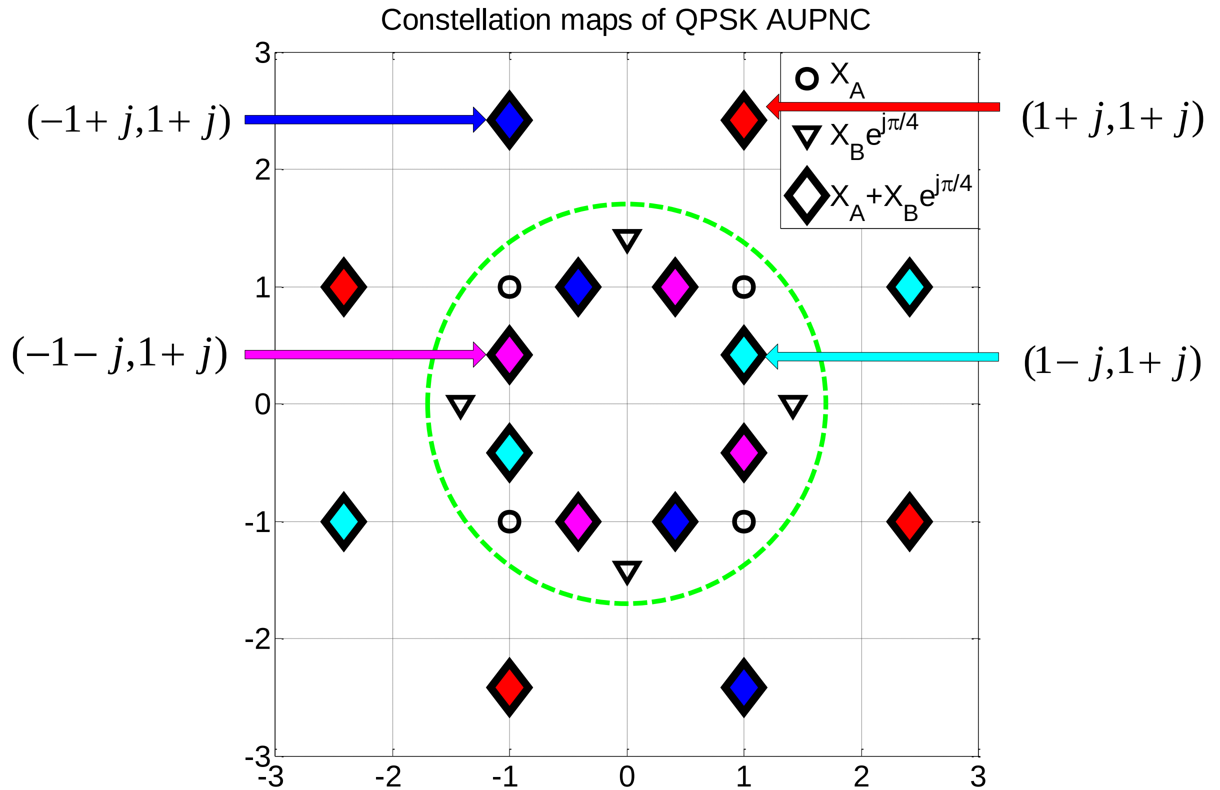
[9] Y. Hao , D. Goeckel , Z. Ding , D. Towsley , K. K. Leung, "Achievable Rates of Physical Layer Network Coding Schemes on the Exchange Channel," *Milcom 2007*

(This paper mentions penalty of 6dB for phase asynchrony in QPSK PNC)

Certainty Propagation with Symbol Misalignment

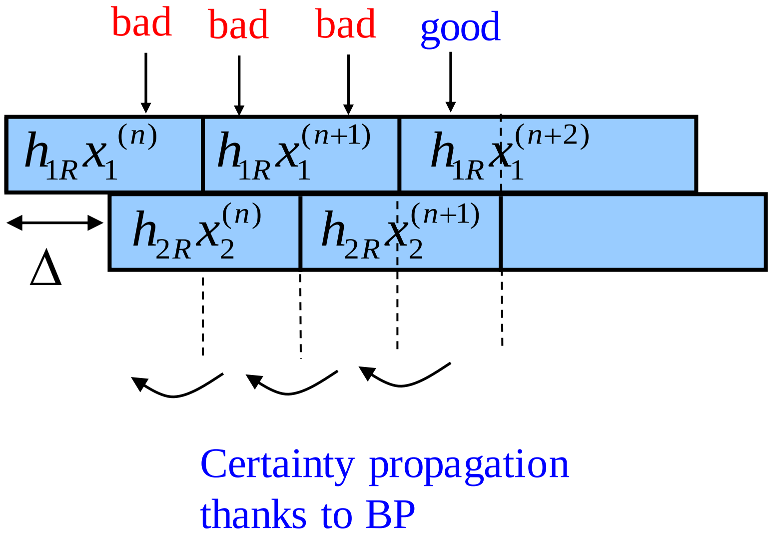


- Same-colored diamonds mapped to the same XOR PNC symbol.
- The outer 8 constellation points are separated further apart than the inner 8 constellation points.



Certainty Propagation with Symbol Misalignment

- Diversity: it pays to have the information of a source symbol straddle in two constellation points.
- "Chain action": it pays even more to have symbols "chained" together in successive constellation points.

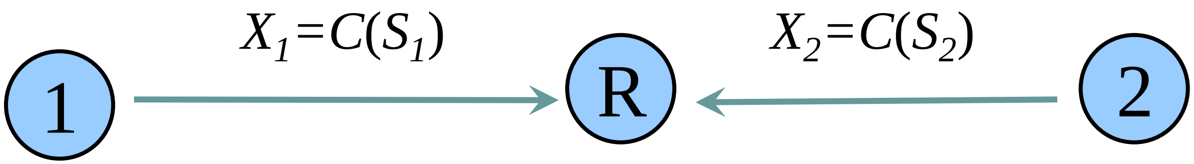


How to Incorporate Channel Coding?

- Two variants: (i) end-to-end; (ii) link-by-link
- Focus on Link-by-link

Channel-coded PNC

Multiaccess (Uplink) Phase:



Reception at relay ($\Delta = 0$ case):

$$y_R[n] = h_{1R}x_1[n] + h_{2R}x_2[n] + w_R[n]$$

$$n = 1, \ldots, N$$

Link-by-Link Channel-coded PNC (1)

Conceptually, channel-coded PNC consists of two parts:

(i) Decoding of $S_R = S_1 \oplus S_2 = (s_1[1] \oplus s_2[1], \ldots, s_1[M] \oplus s_2[M])$ from Y_R

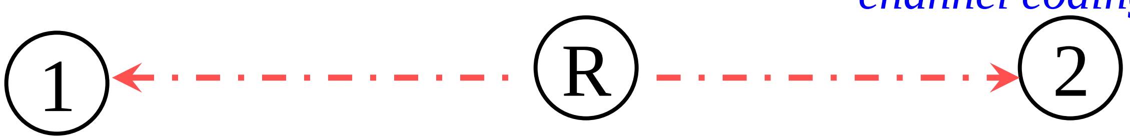
(ii) Computation of $X_R = C(\hat{S}_R)$

(i) $\hat{S}_R = CNC(Y_R)$

*This mapping involves both **C**hannel decoding and **N**etwork **C**oding. Input is N samples, output is M < N symbols when* $\Delta = 0$

(ii) $X_R = C(\hat{S}_R)$

This is similar to conventional channel coding



Key issue: how to do CNC(.)?

Link-by-Link Channel-coded PNC (2)

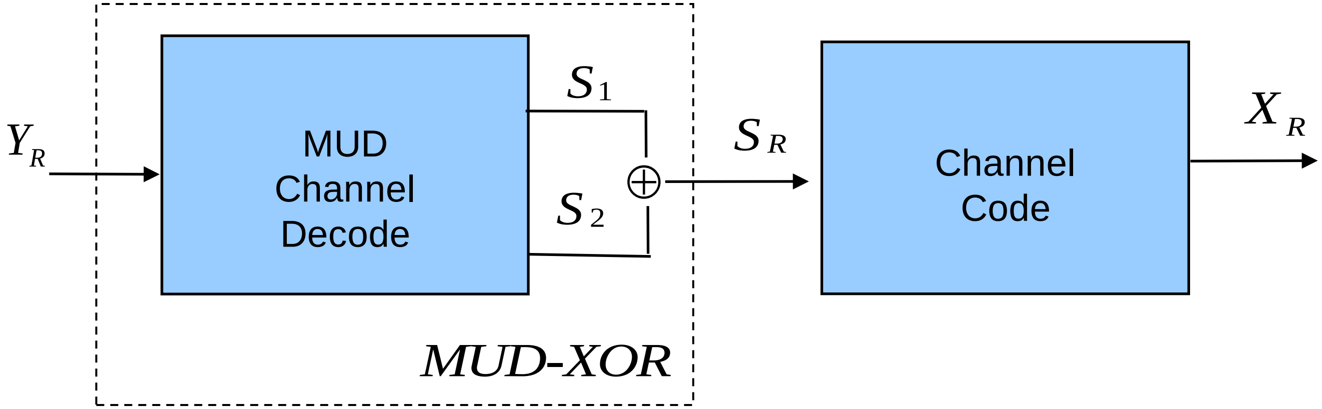
Different methods for mapping $\widehat{S}_R = CNC(Y_R)$

- Disjoint channel-decoding and network-coding operations (two examples in slides to follow)
- Integrated channel-decoding and network-coding operation (one example in a slide to follow)

Link-by-Link Channel-coded PNC (3)

MUD-based channel decoding followed by network coding

Method 1: MUD-XOR



- *MUD Channel Decode could use SIC*
- *Suboptimal, but can achieve exchange capacity of TWRC in low SNR region.*[10]

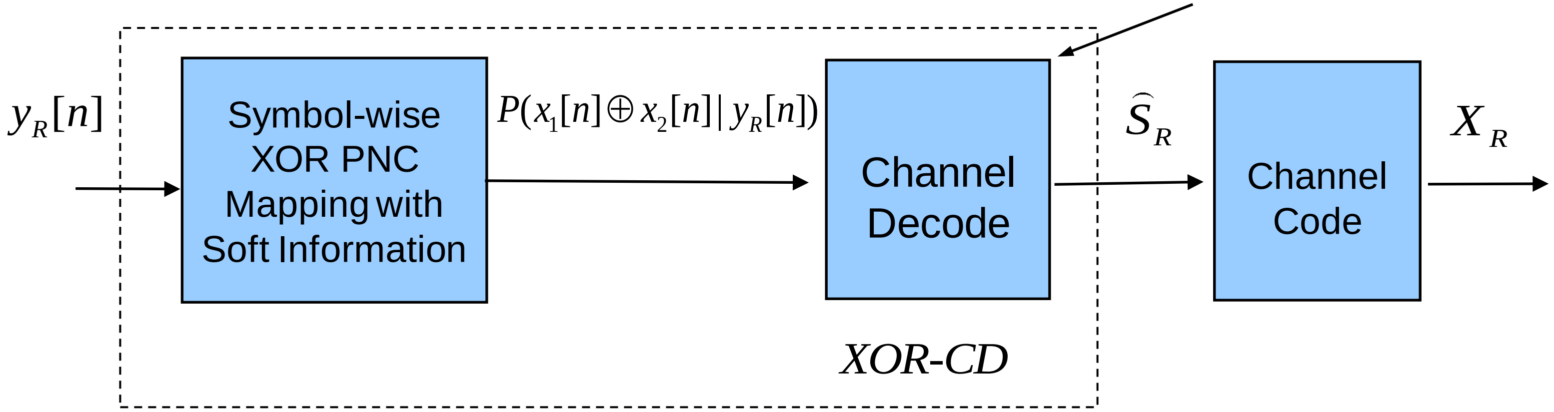
[10] S. Zhang, S. C. Liew, "The capacity of two way relay channels," *Access Network, Springer*, vol. 37, Nov 2009

Link-by-Link Channel-coded PNC (4)

PNC mapping followed by channel decoding

Method 2: XOR-CD [11,12,13]

For linear code $C(\cdot)$, $C(S_R) = C(S_1) \oplus C(S_2) = X_1 \oplus X_2 \Rightarrow S_R = C^{-1}(X_1 \oplus X_2)$



- *Suboptimal but can achieve exchange capacity of TWRC under high SNR region* [14]

[11] S. Zhang and S. C. Liew, "Channel Coding and Decoding in a Relay System Operated with Physical-Layer Network Coding," *JSAC* June 2009. (This paper mentions the suboptimality of the scheme)

[12] F. Rossetto, M. Zorzi "On the Design of Practical Asynchronous Physical Layer network coding", Spawc 2009. (This paper makes use of this scheme with OFDM)

[13] P. Popovski, and H. Yomo, "Physical Network Coding in Two-Way Relay Channel," *ICC* 2007 (This paper studies this scheme)

[14] K Narayanan, MP Wilson, A. Sprintson "Joint Physical Layer Coding and Network Coding for Bi-directional Relaying," *Allerton 2007.*

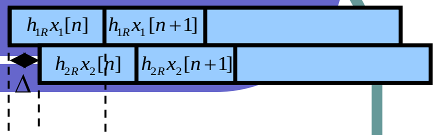
Link-by-Link Channel-coded PNC (7)

<u>Asynchronous integrated channel decoding and network coding</u>

- *Method 4:* Joint *CNC* [15]
  - Extends beyond BPSK
  - Takes care of different channel gains on the two uplinks
  - Deals with phase and symbol misalignments

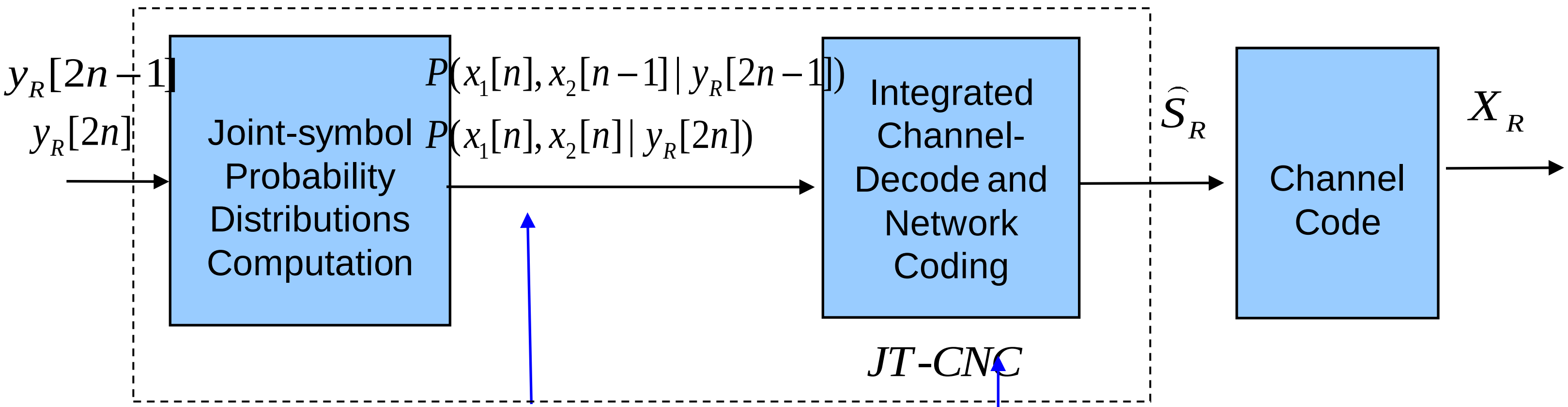
[15] S. C. Liew, L. Lu, *Ongoing Work*

Link-by-Link Channel-coded PNC (8)



Asynchronous integrated channel decoding and network coding

Method 4: *JT-CNC* [15]



Oversampling for symbol offsets

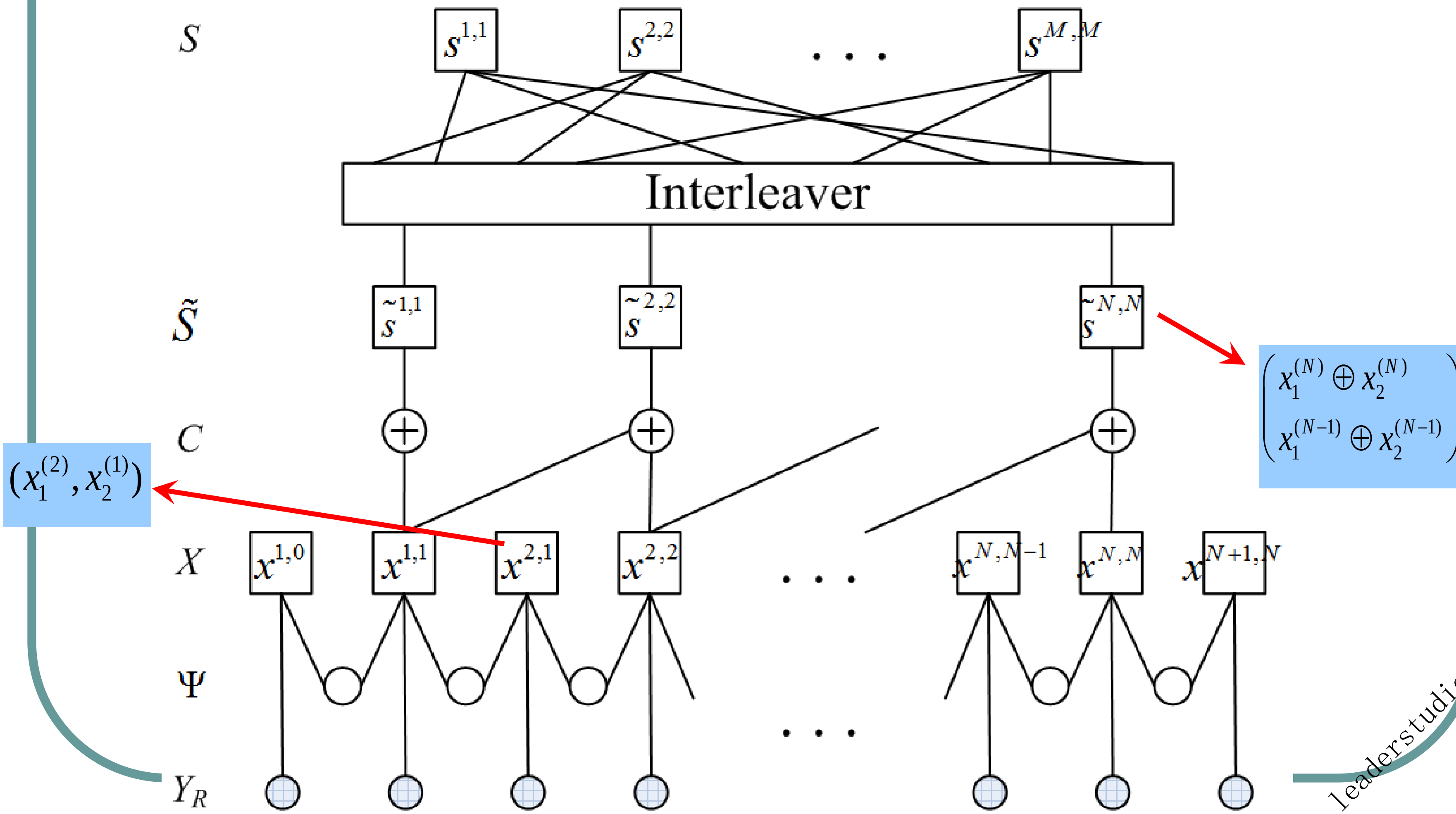
Use belief propagation to find $\Pr(s_1[n], s_2[n] \mid Y_R)$

Then, find $\Pr(s_1[n] \oplus s_2[n] \mid Y_R)$ from $\Pr(s_1[n], s_2[n] \mid Y_R)$

Then, find the maximum likelihood $s_R[n] = s_1[n] \oplus s_2[n]$

Link-by-Link Channel-coded PNC (9)

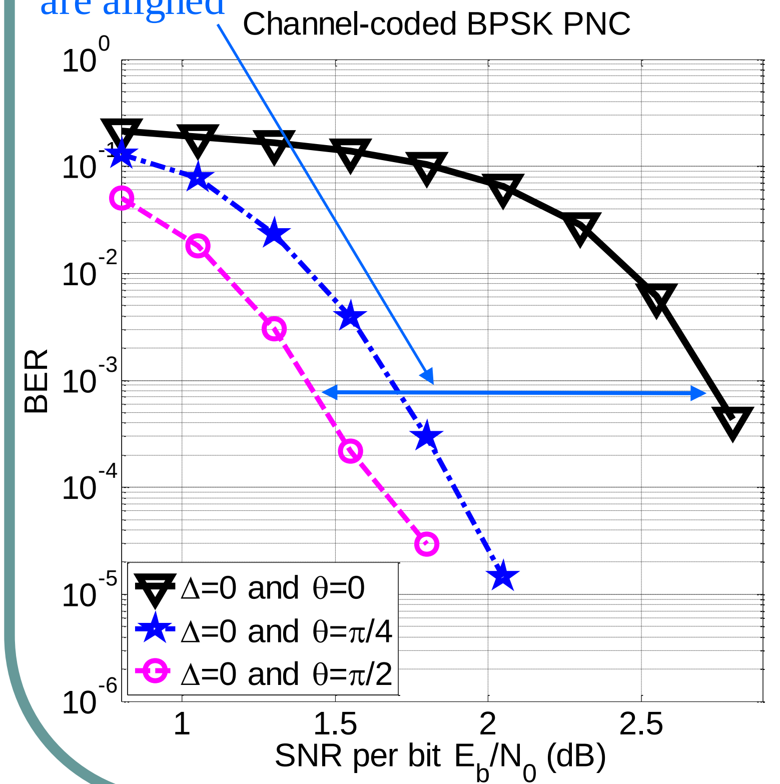
Tanner Graph of *Jt-CNC*



Link-by-Link Channel-coded PNC (9)

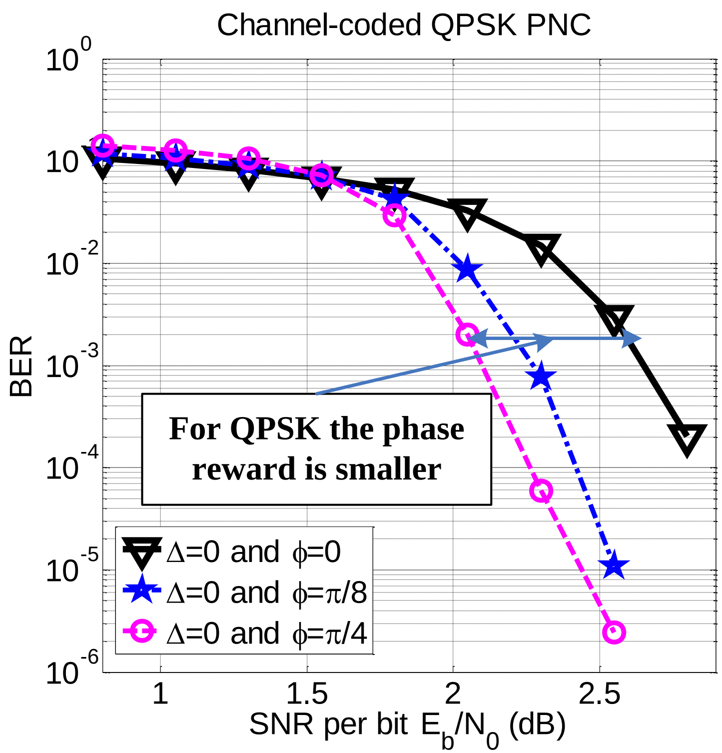
Results of $JT-CNC$ for BPSK

With channel coding, phase asynchrony results in better performance when symbols are aligned

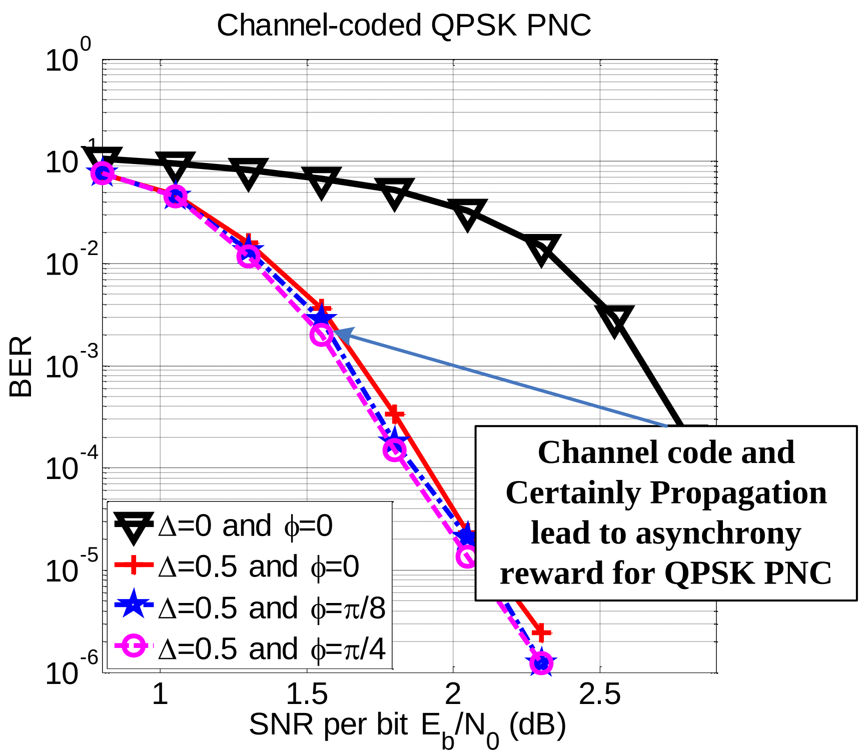


Link-by-Link Channel-coded PNC (10)

Results of *JT-CNC* for QPSK



(a)



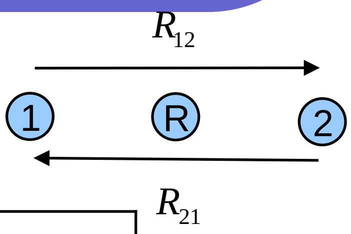
(b)

Outline

- Basic idea of PNC
- Communication-theoretic research
- Information-theoretic research
  - Most work focuses on 3-node linear network: TWRC (two-way relay channel)
  - Significant result to date: use of lattice code can achieve capacity within half bit.
- Network-theoretic research
- Conclusion

Link-by-link Channel-coded PNC Capacity (5)

Achievable symmetric rates (i.e., $R_{12}=R_{21}=R$) under symmetric channels (i.e., $P_{1R}=P_{2R}=P_{R1}=P_{R2}=P$)



P (dB)	0	2	4	6	8	10
$R_{12}^{PNC_{LC}}(t_u^*)$	0.185	0.299	0.424	0.559	0.704	0.856
$R_{12}^{PNC_{MUD}}(t_u^{'})$	0.221	0.294	0.378	0.470	0.569	0.672
R_{12}^{SNC}	0.167	0.228	0.302	0.386	0.478	0.577
R_{12}^{ANC}	0.080	0.131	0.200	0.288	0.396	0.520
R_{12}^{TS}	0.125	0.171	0.227	0.290	0.359	0.432
$U_{12}(1/2)$	0.250	0.343	0.453	0.579	0.717	0.865

3-time slot transmissions → R_{12}^{SNC}

4-time slot transmissions → R_{12}^{TS}

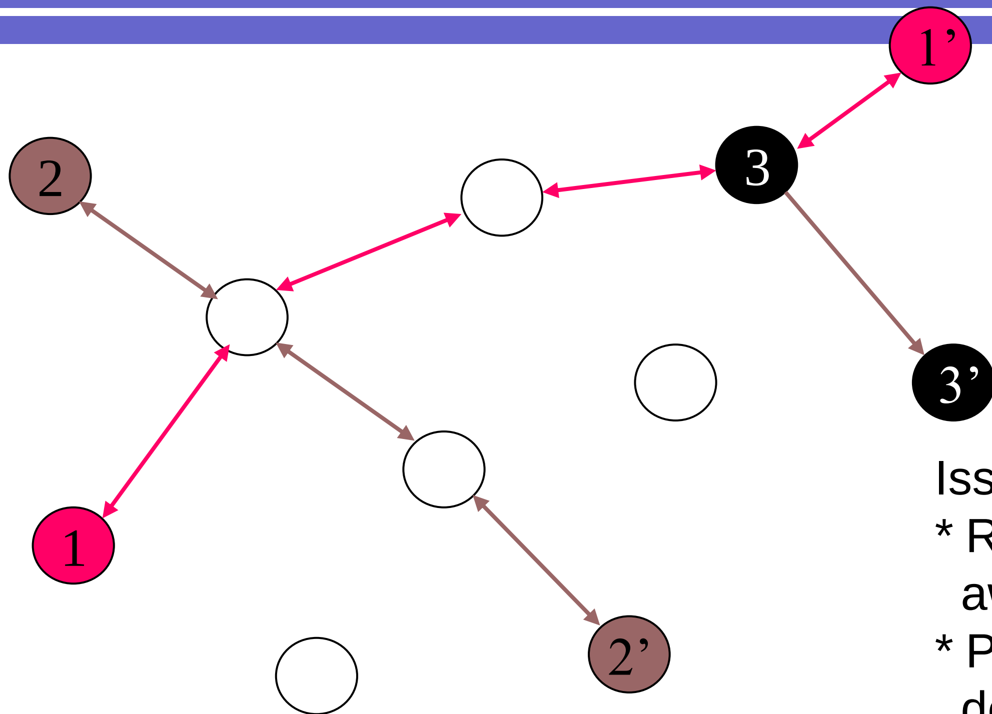
What Else?

- All information-theoretic work to-date assumes symbol level and phase level synchronization
- Half-bit capacity gap is still significant at low SNR region.
  - Lattice-coded PNC is a generalization of XOR-CD
  - How to generalize JT CNC?

Outline

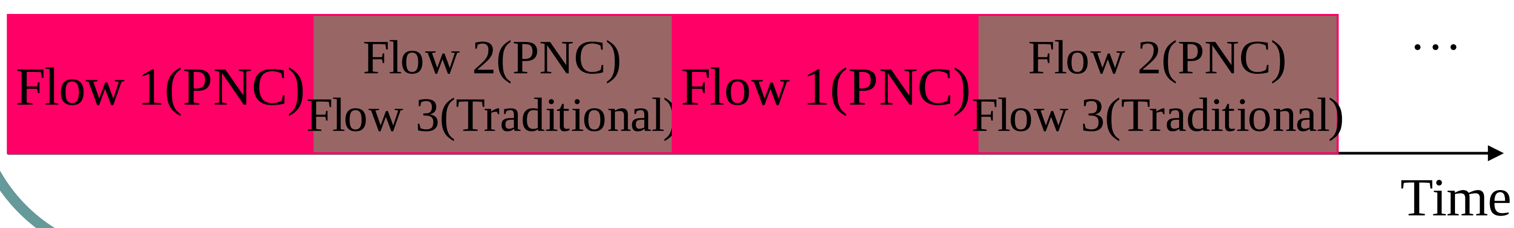
- Basic idea of PNC
- Communication-theoretic research
- Information-theoretic research
- Network-theoretic research
  - Relatively little work
- Conclusion

PNC in General Network



Issues:

* Routing: may not always steer away from interference
* PNC flow construction and decomposition
* Scheduling



Per-flow Throughput Summary [22]

Results from [22]

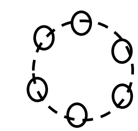
	Traditional Scheduling	Straightforward Network coding	PNC
1-D	$\frac{2}{N}$	$\frac{8}{3N}$	$\frac{4}{N}$
2-D *	$\frac{2}{9\sqrt{N}}$	$\frac{1}{3\sqrt{N}}$	$\frac{2}{3\sqrt{N}}$

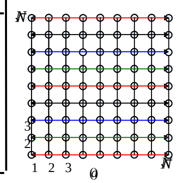
33%

100%

50%

200%





* Assume path loss exponent of 4 and SIR threshold of 10dB

Conclusions and Future Directions

- Communications-Theoretic
  - Embrace asynchrony?
  - Simpler scheme than Jt-CNC that has good performance for all SNR regions?
- Information-Theoretic
  - Outer bound achievable within 1/2 bit with Lattice code. Significant result for high SNR region only.
  - Ultimate capacity region across all SNR regime for TWRC?
- Network-Theoretic
  - Application of PNC in larger-scale networks.
  - Distributed MAC and network algorithms for PNC.
- Implementation work lacking
  - ANC[3]
  - FPNC: we recently completed a prototype of PNC in the frequency domain using OFDM

To Probe Further

- Slides in this talk based on
  "Physical-layer Network Coding: Tutorial, Survey, and Beyond". A 60-page monograph/invited paper.
  (http://arxiv.org/abs/1105.4261)

- Further details on asynchronous PNC:
  "Asynchronous Physical-layer Network Coding".
  (http://arxiv.org/abs/1105.3144)

- Implementation of OFDM-based PNC on soft radio:
  "Implementation of Physical-layer Network Coding".
  (http://arxiv.org/abs/1105.3416)